7

GEAR TRAINS

A *gear train* is composed of two or more gears in mesh for the purpose of transmitting motion (or power) from one shaft to another.

Velocity ratio and train value : The ratio of the angular velocity of the driver gear to that of the driven gear is called *velocity ratio*. The ratio of the angular velocity of the driven gear to that of the driving gear is called the *train value*. Velocity ratio and train value are reciprocal quantities.

Direction of rotation : The direction of gear's rotation is conventionally specified as clockwise or counter clockwise. Clockwise rotation will be regarded as positive and anti-clockwise rotation as negative. When gears mesh externally they rotate in opposite directions and when gears mesh internally they rotate in the same direction as shown in fig. 7.1.

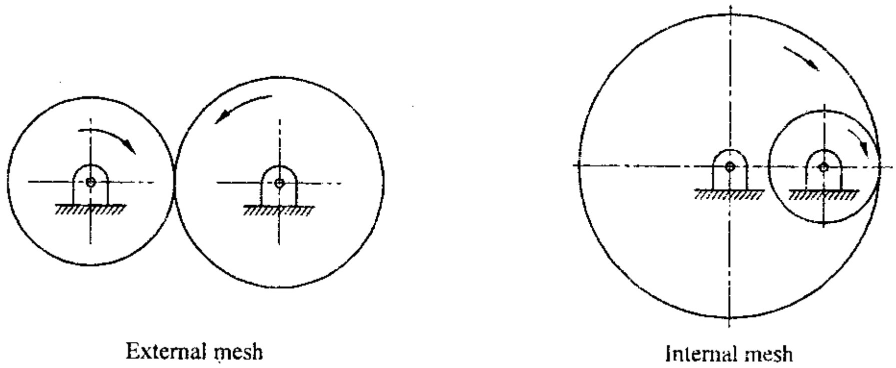

Fig. 7.1

Classification : The gear train may be classified broadly into the following categories.

(1) Simple gear train (2) Compound gear train

(3) Reverted gear train (4) Epicylic gear train

Simple gear train : A simple gear train is one in which each shaft carries only one gear as shown in fig. 7.2. If gear 1 is the driver and gear 3 the driven, then motion is transmitted from 1 to 3 through the intermediate gear 2, known as *idler*. An idler does not affect the train value of the train, it serves only to fill up space and reverse the direction.

Let z_1 = Number of teeth on the driver gear

z_2 = Number of teeth on the intermediate gear

z_3 = Number of teeth on the driven gear

n_1 = Speed of the driver gear
n_2 = Speed of the intermediate gear
n_3 = Speed of the driven gear

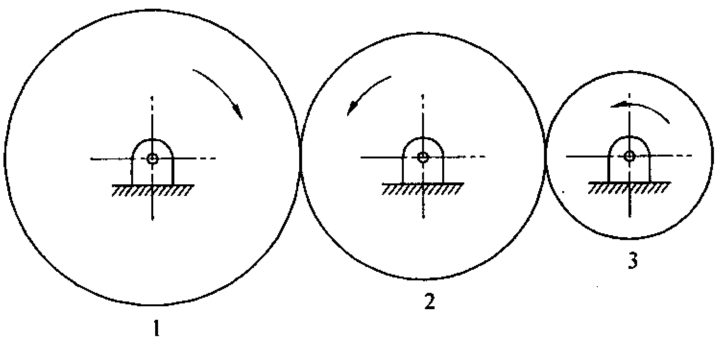

Fig. 7.2

For each pair of meshing gears, the angular velocities vary inversely as the radii and therefore as their number of teeth.

From gears 1 and 2,

Velocity ratio $\frac{n_1}{n_2} = \frac{z_2}{z_1}$ ......(1)

Similarly from gears 2 and 3, $\frac{n_2}{n_3} = \frac{z_3}{z_2}$ ......(2)

Multiplying (1) and (2)

$$\frac{n_1}{n_2} \times \frac{n_2}{n_3} = \frac{z_2}{z_1} \times \frac{z_3}{z_2}$$

or Velocity ratio $\frac{n_1}{n_3} = \frac{z_3}{z_1}$

$$\text{Train value TV} = \frac{n_3}{n_1} = \frac{z_1}{z_3} = \frac{\text{Number of teeth on the driver}}{\text{Number of teeth on the driven}}$$

Compound gear train : A compound gear train is one in which each shaft carries two or more gears and are keyed to it. Fig. 7.3 represents a compound gear train in which gears 2 and 3 constitute compound gear.

From gears 1 and 2 $\frac{n_1}{n_2} = \frac{z_2}{z_1}$ ......(1)

and from gears 3 and 4,

$$\frac{n_3}{n_4} = \frac{z_4}{z_3} \quad \text{......(2)}$$

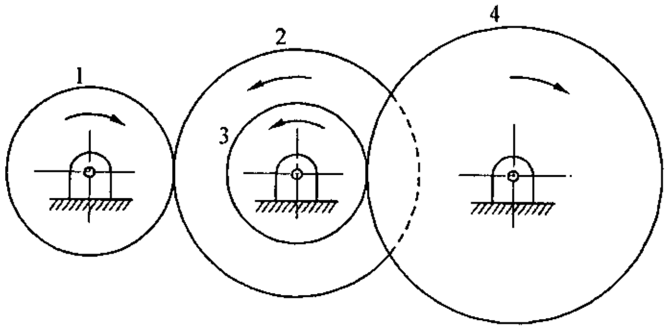

Fig. 7.3

multiplying (1) by (2)

$$\frac{n_1}{n_2} \times \frac{n_3}{n_4} = \frac{z_2}{z_1} \times \frac{z_4}{z_3} \qquad \text{......(3)}$$

as 2 and 3 are compound gears, $n_2 = n_3$, therefore equation (3) becomes

$$\frac{n_1}{n_4} = \frac{z_2}{z_1} \times \frac{z_4}{z_3}$$

$$\therefore \text{TV} = \frac{n_4}{n_1} = \frac{z_1}{z_2} \times \frac{z_4}{z_3} = \frac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on followers}}$$

It is conventional to designate the train value as positive if the driver and the follower rotate in the same direction and negative if they rotate in opposite directions. The advantage of compound gear train is the larger velocity ratio in limited space.

For large speed reduction by using the compound gear train, the following factors have to be taken into account.

1. The total speed reduction required and the largest speed reduction that can be allowed in one step. The maximum speed ratio for a pair of spur gears drive should not usually exceed 5.
2. The space occupied by the gearing depends upon the minimum allowable number of teeth on pinions.
3. The module of the teeth must increase progressively from the high-speed to the low-speed shafts.

Example 7.1

Fig. 7.4 shows a compound gear train. Find the train value

Data :

$z_A = 18, z_B = 72, z_C = 20, z_D = 80, z_E = 22, z_F = 110$

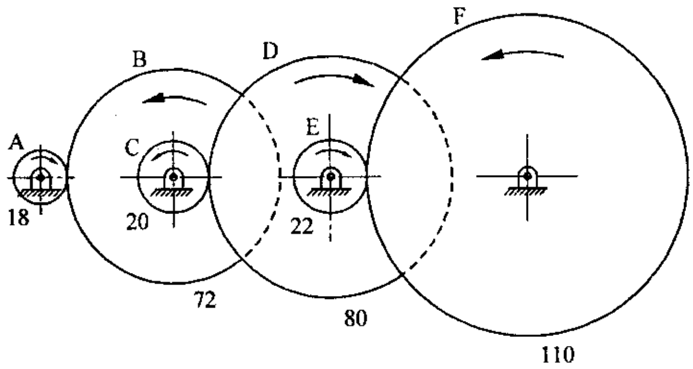

Fig. 7.4

Solution :

Train value $$TV = \frac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on followers}}$$

The first gear A (driver) rotates in clockwise direction and the last gear F (follower) rotates in anticlockwise direction.

$$\therefore \quad TV = -\frac{z_A}{z_B} \times \frac{z_C}{z_D} \times \frac{z_E}{z_F} = -\frac{18}{72} \times \frac{20}{80} \times \frac{22}{110} = -\frac{1}{80}$$

Reverted gear train : A reverted gear train is one in which the first and last gears are on the same axis is as shown in fig. 7.5. In reverted gear trains, the center distances of the two pairs of gears must be the same. Since the diametral pitch is same for both pairs, therefore

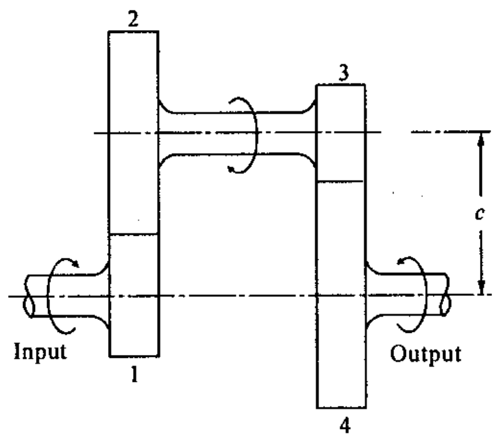

Fig. 7.5

Center distance $c = r_1 + r_2 = r_3 + r_4$

Since number of teeth is proportional to pitch circle radius or $$z_1 + z_2 = z_3 + z_4$$

and $$TV = \frac{n_4}{n_1} = \frac{z_1}{z_2} \times \frac{z_3}{z_4} = \frac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on followers}}$$

Riverted gear trains are used in automotive transmissions, lathe back gears and in clocks.

Example 7.2

Find the train value of a reverted gear train shown in fig. 7.6

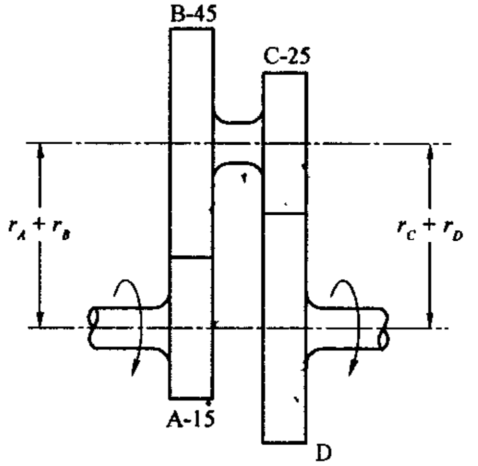

Fig. 7.6

Data:

$z_A = 15$ teeth, $z_B = 45$ teeth, $z_C = 25$ teeth

Solution: In reverted gear train, the center distance of the two pairs of gears must be the same.

i.e., $$r_A + r_B = r_C + r_D$$

Assuming the module is same for all gears, the number of teeth on gears are proportional to their respective pitch circle radius.

i.e., $$z_A + z_B = z_C + z_D$$

$$15 + 45 = 25 + z_D$$

$\therefore$ Number of teeth on gear D, $z_D = 35$

Train value $$TV = \frac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on follower}}$$

The first gear A and the last gear D rotates in the same directions

$$\therefore \quad TV = \frac{z_A}{z_B} \times \frac{z_C}{z_D} = \frac{15}{45} \times \frac{25}{35} = \frac{1}{4.2}$$

Example: 7.3

The speed ratio of each pair in a reverted gear train is to be 4. The module of the gears 1 and 2 is 4 mm and that of gears 3 and 4 is 5 mm. Calculate the suitable number of teeth for the gears if the center distance between shafts is 400 mm.

Data:

$$\frac{n_1}{n_2} = \frac{n_3}{n_4} = 4, \; m_1 = 4 \text{ mm}, \; m_2 = 5 \text{ mm}, \; c = 400 \text{ mm}$$

Solution: *Refer fig. 7.5*

Train value for gears 1-2, $\frac{z_1}{z_2} = \frac{1}{4}$

$\therefore \quad z_2 = 4\,z_1$ ..... (1)

Train value for gears 2-3, $\frac{z_3}{z_4} = \frac{1}{4}, \; \therefore \; z_4 = 4\,z_3$ ..... (2)

Center distance $\quad c = r_1 + r_2 = r_3 + r_4$

i.e., $\quad 400 = \frac{m_1}{2}(z_1 + z_2) = \frac{m_2}{2}(z_3 + z_4)$ $\quad (\because \; r = mz/2)$

$\quad 400 = \frac{4}{2}(z_1 + 4\,z_1) = \frac{5}{2}(z_3 + 4\,z_3)$ ..... (3)

$\therefore$ Number of teeth on gear 1, $z_1 = 40$

Number of teeth on gear 2, $z_2 = 4\,z_1 = 4 \times 40 = 160$

Number of teeth on gear 3, $z_3 = 32$

Number of teeth on gear 4, $z_4 = 4\,z_3 = 4 \times 32 = 128$

Epicyclic gear train : Epicyelic gear train is one in which the axis of one or more gears moves relative to the frame. It is also called as *planetary gear train*. In the arrangement shown in fig. 7.7, the sun gear C is fixed to the frame. The planet gear B, carried by a revolving arm A, rotate not only about their own center but also about the center of the fixed gear.

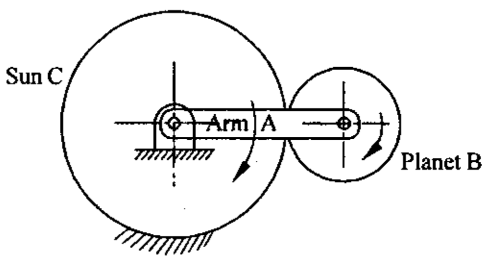

Fig. 7.7

Calculation of train value for planetary gear train is complicated by the movement of the planet gear axis. There are numerous methods available for solving a planetary gear train problem, but only the formula or algebraic method and super position or tabulation method will be discussed here.

Train value by algebraic method

Fig. 7.7 shows an epicycle gear train which is composed of a sun gear C, planet gear B, and the arm A. Assume the gear B is the driver and gear C is the driven member which is free to rotate.

$\therefore$ Train value $\quad TV = \dfrac{z_B}{z_C}$ ...... (1)

The speed of sun gear C relative to the arm A is

$$n_{CA} = n_c - n_A \qquad \text{...... (2)}$$

Also, speed of planet gear B relative to the arm A is

$$n_{BA} = n_B - n_A \qquad \text{...... (3)}$$

Divide the equation (3) by (2) gives

$$\frac{n_{BA}}{n_{CA}} = \frac{n_B - n_A}{n_C - n_A} \qquad \text{...... (4)}$$

Which gives the train value of the planetary gear train.

i.e., $\quad TV = \dfrac{z_B}{z_C} = \dfrac{n_B - n_A}{n_C - n_A}$ ...... (5)

For any planetary gear trains, the equation (5) can be written as

$$TV = \frac{n_L - n_A}{n_F - n_A} \qquad \text{...... (6)}$$

Where $\quad n_F$ = Speed of the first gear in the train

n_L = Speed of the last gear in the train

n_A = Speed of the arm

The required conditions of motion can be applied to the equation (6) to determine the unknown speed.

Example 7.4

In the planetary gear train shown in fig. 7.8, the sun gear B is fixed. Find the speed of gear D, if the arm A rotates at 60 rpm counter-clockwise direction. The number of teeth on B, C, and D are 120, 60 and 40 respectively.

Data :

$n_A = -60$ rpm (counter clockwise), $= 120$, $z_C = 60$, $z_D = 40$

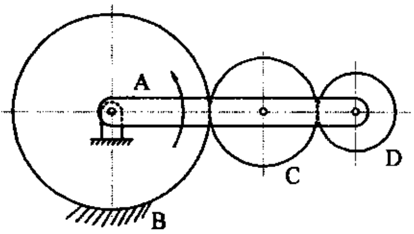

Fig. 7.8

Solution :

Consider the gears B, C, D and the arm A. Let gear B be the first gear and gear D be the last gear of the planetary gear train. The first gear B and the last gear D rotates in the same direction

$\therefore$ Train value $\quad TV = \dfrac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on followers}}$

$= \dfrac{z_B}{z_C} \times \dfrac{z_C}{z_D}$

i.e., $\quad TV = \dfrac{120}{60} \times \dfrac{60}{40} = 3$ ......(1)

Also, train value $\quad TV = \dfrac{n_L - n_A}{n_F - n_A} = \dfrac{n_D - n_A}{n_B - n_A}$

The given conditions are:

Gear B is fixed; $\quad \therefore \quad n_B = 0$

and the speed of arm is $n_A = -60$ rpm

$\therefore \quad TV = \dfrac{n_D + 60}{0 + 60}$ ......(2)

Equating the equations (1) and (2) we get,

$$3 = \frac{n_D + 60}{0 + 60}$$

$\therefore$ Speed of gear D, $n_D =$ 120 rpm (clockwise)

Example 7.5

In the epicyclic gear train shown in fig. 7.9, the annular gear R is fixed and the sun wheel S rotates at 200 rpm clockwise. The number of teeth on the annular gear is 115 and on the sun gear is 45. Assuming the module is same for all gears, determine

i) Number of teeth on the planet gear P, and

ii) Speed and direction of rotation of the arm A.

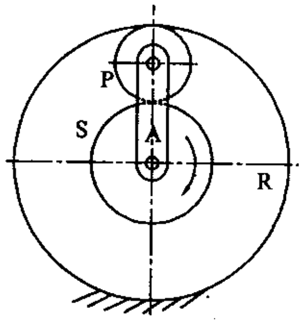

Fig. 7.9

Data :

$n_R = 0$, $n_S = 200$ rpm (clockwise), $z_R = 115$, $z_S = 45$

Solution :

Since the module is same for all meshing gears, the number of teeth is proportional to the pitch circle radius. Therefore the radius of annular gear R interms of sun and planet gear radii is

$$r_R = r_S + 2r_P$$

i.e.,
$$z_R = z_S + 2z_P$$

or
$$115 = 45 + 2z_P$$

$\therefore$ Number of teeth on planet gear P, $z_P = 35$

Consider the gears S, P, R and arm A. Let S be the first gear and R be the last gear of the planetary gear train. The first gear S and the last gear R rotate in the opposite directions.

$\therefore$ Train value
$$TV = -\frac{z_S}{z_P} \times \frac{z_P}{z_R} = -\frac{z_S}{z_R} = -\frac{45}{115}$$

Also,
$$TV = \frac{n_L - n_A}{n_F - n_A} = \frac{n_R - n_A}{n_S - n_A}$$

Substitute $n_R = 0$ and $n_S = 200$ in the above equation, we get

$$\frac{-45}{115} = \frac{0 - n_A}{200 - n_A}$$

$\therefore$ Speed of the arm A, $n_A = 56.25$ rpm (clockwise)

Example 7.6

In the epicyclic gear train shown in fig. 7.10, the arm A, and the gear 5 are driven clockwise at 300 rpm and 100 rpm respectively. The hub of the arm rotates freely on the shaft of the gear 2. The compound gear 3-4 is free to revolve on the arm. The number of teeth on gears 2,3,4, and 5 are 20, 40, 35 and 25 respectively. Determine the speed and direction of the gear 2.

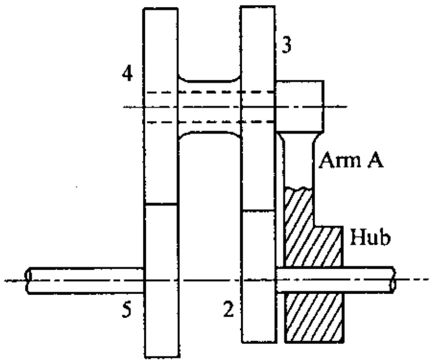

Fig. 7.10

Data :

$n_A = 300$ rpm (clockwise), $n_5 = 100$ rpm (clockwise), $z_2 = 20, z_3 = 40, z_4 = 35, z_5 = 25$

Solution :

Consider the gears 2,3,4,5, and the arm A. Let gear 2 be the first gear and the gear 5 be the last gear of the planetary gear train. The first gear 2 and the last gear 5 rotate in the same direction.

$\therefore$ Train value $\quad TV = \dfrac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on followers}}$

i.e., $\quad TV = \dfrac{z_2}{z_3} \times \dfrac{z_4}{z_5} = \dfrac{20}{40} \times \dfrac{35}{25} = 0.7$

Also, $\quad TV = \dfrac{n_L - n_A}{n_F - n_A} = \dfrac{n_5 - n_A}{n_2 - n_A}$

Substitute the value of $n_A = 300$ rpm, and $n_5 = 100$ rpm in the above equation, we get

$$0.7 = \frac{100 - 300}{n_2 - 300}$$

$\therefore$ Speed of the gear 2, $n_2 = 14.286$ rpm (clockwise)

Example 7.7

An aircraft propeller reduction drive is shown diagrammatically in fig. 7.11. Determine the propeller (arm A) speed in magnitude and direction if the engine runs at 2500 rpm clockwise direction. Gears 2–3 forms a compound gear which in free to rotate on the arm.

Data :

$n_4 = 2500$ rpm (clockwise), $n_1 = 0$, $n_2 = n_3$, $z_1 = 48$, $z_2 = 28$, $z_3 = 46$, $z_4 = 124$

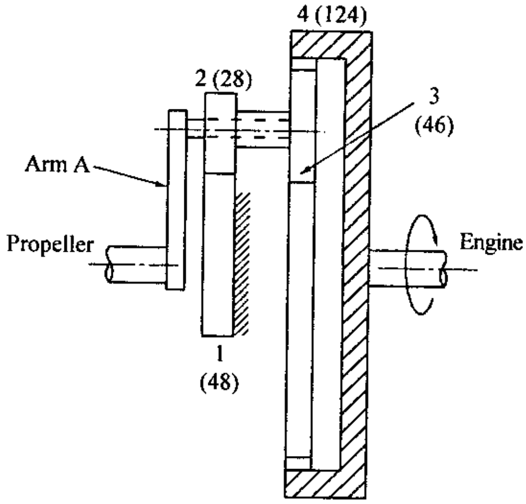

Fig. 7.11

Solution :

Consider the gears 1,2,3,4 and arm A. Let gear 1 be the first gear and gear 4 be the last gear of the planetary gear train. The first and the last gear rotate in opposite directions.

$\therefore$ Train value $\quad TV = \dfrac{\text{Product of number of teeth on dirvers}}{\text{Product of number of teeth on followers}}$

$$= -\frac{z_1}{z_2} \times \frac{z_3}{z_4} = -\frac{48}{28} \times \frac{46}{124}$$

Also, $\quad TV = \dfrac{n_L - n_A}{n_F - n_A} = \dfrac{n_4 - n_A}{n_1 - n_A}$

Substitute the values of $n_4 = 2500$ rpm, $n_1 = 0$ in the above equation, we get

$$-\frac{48}{28} \times \frac{46}{124} = \frac{2500 - n_A}{0 - n_A}$$

$\therefore$ Speed of arm A, $n_A = 1528.17$ rpm (clockwise)

Example 7.8

In the planetary reduction unit shown in fig. 7.12, gear 2 runs at 200 rpm clockwise. Gears 3 and 4 forms a compound gear and is free to rotate on the arm. Determine

i) Number of teeth on the internal gear 1, and

ii) Speed and direction of rotation of gear 5.

Data :

$n_2 = 200$ rpm (clockwise), $n_1 = 0$ (fixed), $z_2 = 25, z_3 = 35, z_4 = 20, z_5 = 40$

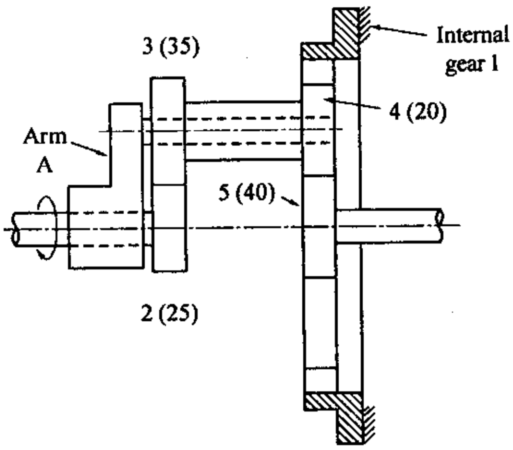

Fig. 7.12

Solution :

Since the module is same for all meshing gears, the number of teeth is proportional to the pitch circle radius. Therefore the pitch circle radius of the internal gear 1 in terms of the pitch circle radii of gears 4 and 5 is

$$r_1 = r_5 + 2r_4$$

∴ Number of the teeth on the internal gear 1 is

$$z_1 = z_5 + 2z_4 = 40 + 2 \times 20 = 80 \text{ teeth}$$

Consider the gears 2, 3, 4, 1 and arm A. Let gear 2 be the first gear and the internal gear 1 be the last gear of the planetary gear train. The first gear 2 and the last gear 1 rotate in the opposite directions.

∴ Train value $\quad TV = \dfrac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on followers}}$

i.e, $\quad TV = -\dfrac{z_2}{z_3} \times \dfrac{z_4}{z_1} = -\dfrac{25}{35} \times \dfrac{20}{80}$

Also, $\quad TV = \dfrac{n_L - n_A}{n_F - n_A} = \dfrac{n_1 - n_A}{n_2 - n_A}$

Substitute the values of $n_1 = 0$, and $n_2 = 200$ rpm in the above equation, we get

$$-\frac{25}{35} \times \frac{20}{80} = \frac{0 - n_A}{200 - n_A}$$

∴ Speed of the arm A, $n_A = 30.303$ rpm (clockwise)

In the above planetary gear train, the gear 5 is not included. Now considering the gears 2, 3, 4, 5, and arm A. Let gear 2 be the first gear and gear 5 be the last gear and they rotate in the same direction.

$\therefore$ Train value $\quad TV = \dfrac{\text{Product of number of teeth on dirvers}}{\text{Product of number of teeth on followers}}$

$$= \frac{z_2}{z_3} \times \frac{z_4}{z_5} = \frac{25}{35} \times \frac{20}{40}$$

Also, $\quad TV = \dfrac{n_L - n_A}{n_F - n_A} = \dfrac{n_5 - n_A}{n_2 - n_A}$

Substitute the values of $n_2 = 200$ rpm, and $n_A = 30.303$ rpm in the above equation, we get

$$\frac{25}{35} \times \frac{20}{40} = \frac{n_5 - 30.303}{200 - 30.303}$$

$\therefore$ Speed of the gear 5, $n_5 = 90.909$ rpm (clockwise)

Example 7.9

Fig. 7.13 shows a compound epicyclic gear train. If the shaft P is driven at 500 rpm while the annulus 6 rotates at 500 rpm in the opposite direction, determine the speed and the direction of the shaft A. The internal gear 3 and the external gear 4 forms a compound gear and is free to rotate on the arm A. The number of teeth on gears 1, 3, 4, and 6 are 30, 80, 25 and 75 respectively.

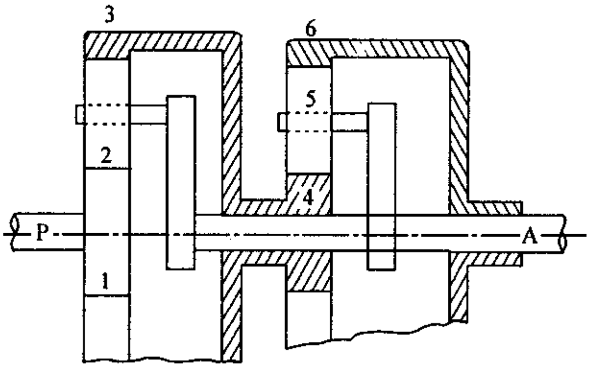

Fig. 7.13

Data :

$n_P = n_1 = 500$ rpm (clockwise), $n_6 = -500$ rpm (anti-clockwise), $z_1 = 30, z_3 = 80, z_4 = 25, z_6 = 75$

Solution :

Consider the gears 1, 2, 3, 4, 5, 6 and the arm A. Let the gear 1 be the first gear and gear 6 be the last gear of the planetary gear train. The first gear 1 and the last gear 6, both rotate in the same direction.

$\therefore$ Train value $\quad TV = \dfrac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on followers}}$

$$= \frac{z_1}{z_2} \times \frac{z_2}{z_3} \times \frac{z_4}{z_5} \times \frac{z_5}{z_6} = \frac{z_1}{z_3} \times \frac{z_4}{z_6} = \frac{30}{80} \times \frac{25}{75} = 0.125$$

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Also, $$TV = \frac{n_L - n_A}{n_F - n_A} = \frac{n_6 - n_A}{n_1 - n_A}$$

Substitute the values of $n_1 = 500$ rpm, $n_6 = -500$ rpm in the above equation, we get

$$0.125 = \frac{-500 - n_A}{500 - n_A}$$

$\therefore$ Speed of the arm A, $n_A = -642.86$ rpm (anticlockwise)

The direction of rotation is same as the gear 6

Example 7.10

A bevel gear differential is shown in fig. 7.14. Gears 3-4 and 5-6 are compound gears and they rotate freely on the arm A. The speed of the arm A is 100 rpm in anticlockwise direction and that of shaft B is 1200 rpm in clockwise direction. The number of teeth on gears 2, 3, 4, 5, 6 and 7 are respectively 30, 60, 40, 80 35, and 25. Determine the speed and the direction of shaft C.

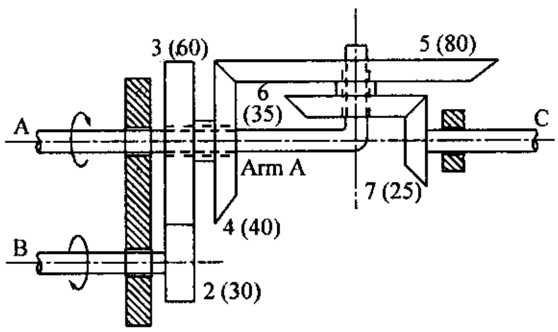

Fig. 7.14

Data :

$n_A = -100$ rpm (anticlockwise), $n_B = n_2 = 1200$ rpm (clockwise), $z_2 = 30$, $z_3 = 60$, $z_4 = 40$, $z_5 = 80$, $z_6 = 35$, $z_7 = 25$

Solution :

Consider the gears 2 and 3,

$$\frac{n_3}{n_2} = \frac{z_2}{z_3}$$

i.e, $$\frac{n_3}{1200} = \frac{30}{60}$$

$\therefore$ Speed of the gear $n_3 = 600$ rpm (anticlockwise direction)

Gears 3 and 4 are compound gears.

$$\therefore n_3 = n_4 = -600 \text{ rpm}$$

Consider the gears 4, 5, 6, 7, and arm A. Let gear 4 be the first gear, and gear 7 be the last gear of the planetary gear train. The first and the last gears rotate in the opposite directions.

$$\therefore \text{Train value} \quad TV = \frac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on followers}}$$

$$= \frac{-z_4}{z_5} \times \frac{z_6}{z_7} = -\frac{40}{80} \times \frac{35}{25} = -0.7$$

$$\text{Also, train value} \quad TV = \frac{n_L - n_A}{n_F - n_A} = \frac{n_7 - n_A}{n_4 - n_A}$$

Substitute the values of n_4 and n_A in the above equation, we get

$$-0.7 = \frac{n_7 + 100}{-600 + 100}$$

$\therefore$ Speed of the last gear $n_7 = n_c = 250$ rpm (clockwise)

Train value of epicyclic gear train by tabular method

Consider the epicyclic gear train as shown in fig. 7.15. Let us assume the arm A is fixed and all the other gears are free to rotate. When gear B makes one revolution clockwise, the gear C will make $-\frac{z_B}{z_C} \times 1$ revolutions anti-clockwise. The statement of relative motion

i.e., Direction sign $\times \frac{\text{Number of teeth on driver}}{\text{Number of teeth on follower}} \times$ Speed of driver is entered in the first row of the table.

Secondly if the wheel B makes x revolutions, then the gear C will make $-\frac{z_B}{z_C}x$ revolutions. In other words multiply the each motion entered in the first row by x. This statement is entered in the second row of the table.

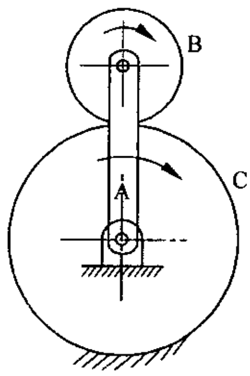

Fig. 7.15

Thirdly, each element of an epicylic train is given y revolutions. In other words add y to each motion entered in the second row to get the third row. The required conditions of motion may be applied to the third row to find the unknown speed.

Condition	Arm-B	Gear B-z_B	Gear C-z_C
Fix the arm A. Give one clockwise revolution to B.	0	+1	$-\frac{z_B}{z_C} \times 1$
Multiply by x	0	x	$-\frac{z_B}{z_C} x$
Add y	y	$y + x$	$y - \frac{z_B}{z_C} x$

Torque in an epicylic gear train : (Refer fig. 7.16)

Let M_{ti} = Input torque

M_{to} = Output or resisting torque

M_{tf} = Holding of fixing torque

If there is no acceleration, the net torque on the gear train must be zero.

$$\therefore \quad M_{ti} + M_{to} + M_{tf} = 0 \qquad \text{..... (1)}$$

$$F_i r_i + F_o r_o + F_f r_f = 0$$

where F_i, F_0, and F_f are the corresponding externally applied forces at radii r_i, r_o, and r_f respectively.

If there is no friction loss, the net energy dissipated by the train must be zero.

$$\therefore \quad M_{ti}\,\omega_i + M_{to}\,\omega_o + M_{tf}\,\omega_f = 0 \qquad \text{..... (2)}$$

where ω_i, ω_0, ω_f are the uniform angular velocities of the input, output and annulus or fixed gear

Input torque $\quad M_{ti} = \dfrac{1000\,P}{\omega_i} = \dfrac{1000\,P \times 60}{2\pi n_i} \qquad$ ..... (3)

where P is the input power to the train.

For a fixed member $\omega_f = 0$

Therefore equation (2) becomes, $M_{ti}\,\omega_i + M_{to}\,\omega_o = 0$

$\therefore$ Output torque $M_{to} = -\dfrac{M_{ti}\,\omega_i}{\omega_o} \qquad$ ..... (4)

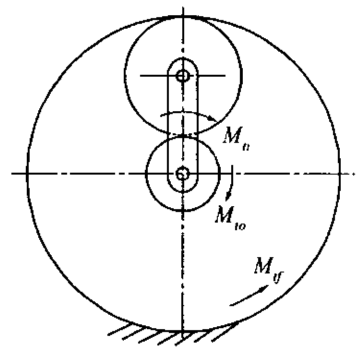

Fig. 7.16

From equation (1), the fixing torque $M_{tf} = -(M_{ti} + M_{to})$

$$= -\left(M_{ti} - \frac{M_{ti}\,\omega_i}{\omega_o}\right) = M_{ti}\left(\frac{\omega_i}{\omega_o} - 1\right)$$

$$\therefore \text{ Holding torque } M_{tf} = M_{ti}\left(\frac{n_i}{n_o} - 1\right) \qquad \text{..... (5)}$$

Example 7.11

Fig. 7.17 shows an epicyclic gear train arrangement. The wheel F is fixed. Two compound gears BC and DE are mounted to the pins P and Q respectively. If the arm R makes one revolution per second clockwise, determine the speed and the direction of rotation of wheel A and G.

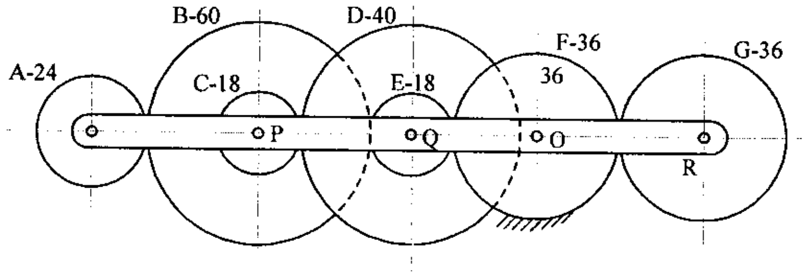

Fig. 7.17

Data :

$n_F = 0$, $n_R = 1$ rps (clockwise), $z_A = 24$, $z_B = 60$, $z_C = 18$, $z_D = 40$, $z_E = 18$, $z_F = 36$, $z_G = 36$

Solution :

Condition	Arm R	A-24	Compound gear		Compound gear		F - 36	G - 36
			B - 60	C - 18	D - 40	E - 18		
Fix the arm R. Give one clockwise revolution to A	0	+1	$\frac{-24}{60}\times 1$	$\frac{-24}{60}\times 1$	$\frac{18}{40}\times\frac{24}{60}$	$\frac{18}{40}\times\frac{24}{60}$	$-\frac{18}{36}\times\frac{18}{40}\times\frac{24}{60}$	$\frac{36}{36}\times\frac{18}{36}\times\frac{18}{40}\times\frac{24}{60}$
Multiply by x	0	x	$-\frac{2}{5}x$	$-\frac{2}{5}x$	$\frac{9}{50}x$	$\frac{9}{50}x$	$-\frac{9}{100}x$	$\frac{9}{100}x$
Add y	y	$y + x$	$y-\frac{2}{5}x$	$y-\frac{2}{5}x$	$y+\frac{9}{50}x$	$y+\frac{9}{50}x$	$y-\frac{9}{100}x$	$y+\frac{9}{100}x$

Arm R makes 1 rps counter clockwise, from the third row of table, $y = -1$ ......(1)

Wheel F is fixed, therefore, $y - \frac{9}{100}x = 0$ ......(2)

substituting the value of $y = -1$ in equation (2) we get

$$-1 - \frac{9}{100}x = 0$$

$$\text{or} \quad x = -\frac{100}{9}$$

Speed of wheel A, $n_A = y + x = -1 - \dfrac{100}{9} = -12.11$ rps (counter clockwise)

Speed of wheel G, $n_G = y + \dfrac{9}{100}\,x = -1 - \dfrac{9}{100} \times \dfrac{100}{9} = -2$ rps (counter clockwise)

Example 7.12

Fig. 7.18 shows an epicyclic gear train. Wheel E is fixed and wheels C and D are integrally cast and mounted on the same pin. If arm A makes 1 revolution per second (counter clockwise), determine the speed and direction of the wheels B and F. **(VTU - March 2001)**

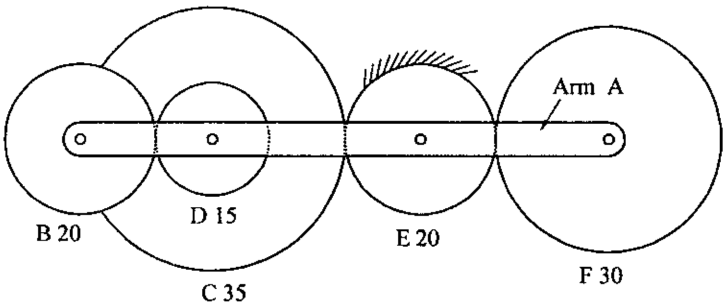

Fig. 7.18

Data:

$z_B = 20$ teeth, $z_C = 35$ teeth, $z_D = 15$ teeth, $z_E = 20$ teeth, $z_F = 30$ teeth, $n_A = -1$ rps, $n_E = 0$

Condition	Arm A	B - 20	Compound Gear		E - 20	F - 30
			D - 15	C - 35		
Fix the arm A. Give one clockwise rotation to gear B.	0	1	$-\dfrac{20}{15}$	$-\dfrac{20}{15}$	$\dfrac{35}{20} \times \dfrac{20}{15}$	$-\dfrac{20}{30} \times \dfrac{35}{20} \times \dfrac{20}{15}$
M	0u	1 x	$-\dfrac{4}{3}\,x$	$-\dfrac{4}{3}\,x$	$\dfrac{7}{3}\,x$	$-\dfrac{14}{9}\,x$
Add y	y	$y + x$	$y - \dfrac{4}{3}\,x$	$y - \dfrac{4}{3}\,x$	$y + \dfrac{7}{3}\,x$	$y - \dfrac{14}{9}\,x$

The gear E is fixed. From the third row of the table, $n_E = y + \dfrac{7}{3}\,x = 0$ ..... (1)

Arm A makes one rps counter clockwise, i.e., $n_A = y = -1$ ..... (2)

Solving the above two equations, we get $x = \dfrac{3}{7}$

Speed of gear B, $n_B = y + x$

$$= -1 + \frac{3}{7} = -0.5714 \text{ rps (counter clockwise)}$$

Speed of gear F, $n_F = y - \frac{14}{9} x$

$$= -1 - \frac{14}{9} \times \frac{3}{7} = -1.67 \text{ rps (counter clockwise)}$$

Example 7.13

A compound epicyclic gear is shown diagrammatically in fig. 7.19. The gears A,D and E are free to rotate on the axis P. The compound gear BC rotate on the axis Q at the end of the arm R. All the gears have equal pitch. The numbers of external teeth on the gears A, B and C are 18, 45 and 21 respectively. The gears D and E are annular gears. The gear A rotates at 90 rpm in the anti-clockwise direction and the gear D rotates at 450 rpm clockwise. Find the speed and direction of the arm R and the gear E.

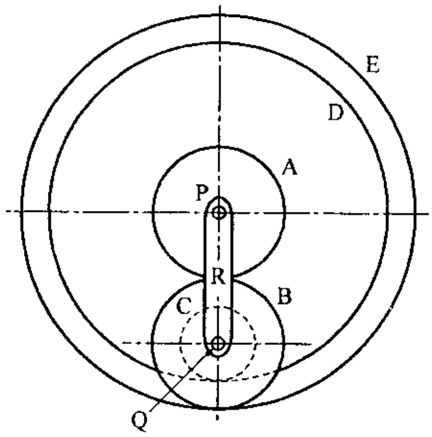

Fig. 7.19

Data :

$z_A = 18$ teeth, $z_B = 45$ teeth, $z_C = 21$ teeth, $n_A = -90$ rpm (anti clockwise), $n_D = 450$ rpm (clockwise)

Solution :

Since the diameteral pitch for meshing gears being same, number of teeth is proportional to pitch circle radius. Therefore the number of teeth on D is.

$$z_D = z_A + z_B + z_C = 18 + 45 + 21 = 84 \text{ teeth}$$

Similarly $$z_E = z_A + 2\, z_B = 18 + 2 \times 45 = 108 \text{ teeth}$$

Condition	Arm R	A -18	Compound gear		D - 84	E - 108
			B - 45	C - 21		
Fix the arm R. Give one clockwise revolution to A	0	+1	$-\frac{18}{45}\times 1$	$-\frac{18}{45}\times 1$	$-\frac{21}{84}\times\frac{18}{45}$	$-\frac{45}{108}\times\frac{18}{45}$
Multiply by x	0	x	$-\frac{18}{45}x$	$-\frac{18}{45}x$	$-\frac{21}{84}\times\frac{18}{45}x$	$-\frac{18}{108}x$
Add y	y	$y+x$	$y-\frac{18}{45}x$	$y-\frac{18}{45}x$	$y-\frac{21}{84}\times\frac{18}{45}x$	$y-\frac{18}{108}x$

Gear A rotates 90 rpm in anti-clockwise direction. From the third row of the table,

$$y+x = -90 \qquad \text{......(1)}$$

Gear D rotates 450 rpm clockwise direction

$$\therefore\ y-\frac{21}{84}\times\frac{18}{45}x = 450 \qquad \text{......(2)}$$

$$(1)-(2)\text{ gives, } 1.1\,x = -540$$

$$\therefore\quad x = -490.9$$

Substituting the value of x in equation (1) we get,

$$y-490.9 = -90$$

$$\therefore\quad y = 490.9-90 = 400.9$$

The speed of the arm R $= y = 400.9$ rpm (clockwise)

The speed of gear E $= y - x$

$$= 400.9-\frac{18}{108}\times(-490.9) = 482.72 \text{ rpm (clockwise)}$$

Example 7.14

An epicyclic gear train shown in fig. 7.20 consists of two sun wheels A and D with 80 and 48 teeth respectively. The sun wheels are engaged with a compound planet wheels B-C. The number of teeth on gear C is 72. Find the speed and direction of wheel D when the wheel A is fixed. The arm R makes 200 rpm clockwise. If 2 kW is transmitted by the arm, what is the torque required to hold the sun wheel A?

Data:

$z_A = 80,\ z_D = 48,\ z_C = 72,\ n_A = 0,\ n_R = 200$ rpm (clockwise), $p = 2$ kW

From the fig. 7.20, $r_D + r_C = r_A + r_B$

i.e., $z_D + z_C = z_A + z_B$

$$48 + 72 = 80 + z_B$$

The number of teeth on B, $z_B = 40$

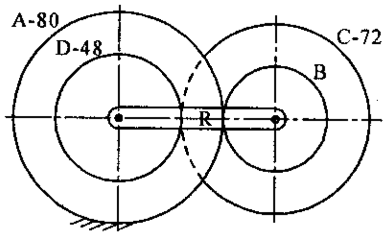

Fig. 7.20

Condition	Arm R	A -80	Compound gear		D - 48
			B - 40	C - 72	
Fix the arm R. Give one clockwise revolution to A	0	+1	$-\frac{80}{40}\times 1$	$-\frac{80}{40}\times 1$	$\frac{72}{48}\times\frac{80}{40}$
Multiply by x	0	x	$-2x$	$-2x$	$3x$
Add y	y	$y+x$	$y-2x$	$y-2x$	$y+3x$

The arm R makes 200 rpm clockwise. From the third row of the table, $y = 200$ ..... (1)

The wheel A is fixed. i.e., $y + x = 0$ ..... (2)

Solving equations (1) and (2) we get, $x = -200$

Speed of the wheel D, $n_D = y + 3x = 200 + 3 \times (-200) = -400$ rpm (CCW)

Speed of wheel A, $n_A = y + x = 200$

Input torque on the arm R, $M_{tR} = \dfrac{1000P}{\omega_R} = \dfrac{1000P \times 60}{2\pi n_R}$

$$= \frac{1000 \times 2 \times 60}{2\pi \times 200} = 95.493 \text{ N m}$$

The energy equation is, $M_{tR}\, n_R + M_{tD}\, n_D + M_{tA}\, n_A = 0$

Output torque on D, $M_{tD} = -\dfrac{M_{tR}\, n_D}{n_D}$ $(\because\ n_A = 0)$

$$= \frac{-95.493 \times 200}{-400} = 47.7465 \text{ N m}$$

The torque equation is, $M_{tR} + M_{tD} + M_{tA} = 0$

$95.493 + 47.7465 + M_{tA} = 0$

Torque required to hold the sun wheel A, $M_{tA} = -143.2395$ N m

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Example 7.15

The fig. 7.21 shows an epicyclic gear train where the arm A is the driver and the annular gear D is the follower. The wheel D has 112 teeth and B has 48 teeth. B runs freely on P and D is separately driven. The arm A runs at 100 rpm and the wheel D at 50 rpm in same direction, find the torque on B if A receives 7.5 kW.

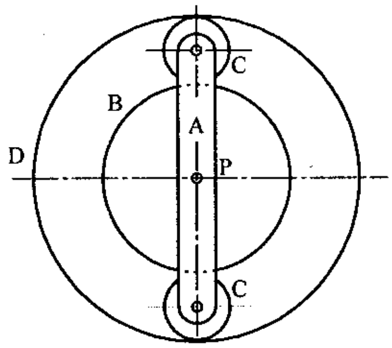

Fig. 7.21

Data :

$z_D = 112$ teeth, $z_B = 48$ teeth, $n_A = 100$ rpm, $n_D = 50$ rpm, $P = 7.5$ kW

Solution :

Number of teeth is proportional to the pitch circle radius

$$\therefore z_D = z_B + 2\,z_C$$

The number of teeth on C, $z_C = \dfrac{z_D - z_B}{2} = \dfrac{112 - 48}{2} = 32$ teeth

Condition	Arm - A	B - 48	C - 32	D - 112
Fix the arm A. Give one clockwise revolution to B	0	+1	$-\frac{48}{32}\times 1$	$-\frac{32}{112}\times\frac{48}{32}$
Multiply by x	0	x	$-\frac{3}{2}x$	$-\frac{3}{7}x$
Add y	y	$y + x$	$y-\frac{3}{2}x$	$y-\frac{3}{7}x$

Speed of arm A is 100 rpm. From the third row of table, $y = 100$ ......(1)

Speed of D is 50 rpm, $\therefore\ y - \frac{3}{7}x = 50$ ......(2)

Substituting the value of y in equation (2) we get.

$$100 - \frac{3}{7}x = 50$$

or $$x = \frac{50 \times 7}{3} = 116.67$$

Speed of wheel B, $n_B = y + x = 100 + 116.67 = 216.67$ rpm

Input torque at A $M_{tA} = \frac{1000\,P}{\omega_A} = \frac{1000\,P \times 60}{2\pi n_A}$

$$= \frac{1000 \times 7.5 \times 60}{2\pi \times 100} = 716.19 \text{ N m}$$

The net torque on the gear train is $M_{tA} + M_{tB} + M_{tD} = 0$

$\therefore$ Torque on gear D, $M_{tD} = -(M_{tA} + M_{tB})$

The energy equation is, $M_{tA}\,n_A + M_{tB}\,n_B + M_{tD}\,n_D = 0$

i.e ., $M_{tA}\,n_A + M_{tB}\,n_B - (M_{tA} - M_{tB})\,n_D = 0$

i.e., $716.19 \times 100 + 216.67\,M_{tB} - (716.19 - M_{tB}) \times 50 = 0$

$\therefore$ Torque on gear B, $M_{tB} = 214.85$ N m

Example 7.16

In an epicyclic gear train of the sun and planet type of fig. 7.22, the pitch diameter of the internally toothed ring D is to be as nearly as possible 228 mm, and the module is 4 mm. When the ring is stationary, the spider A which carries three planet wheel C of equal size is to make one revolution for every five revolutions of the driving spindle carrying the sun wheel B. Determine suitable number of teeth for all the wheels and the exact pitch circle diameter of the ring D.

If a torque of 30 N m is applied to the sun wheel B, what torque will be required to keep the ring stationary? **(VTU - Jan 2007)**

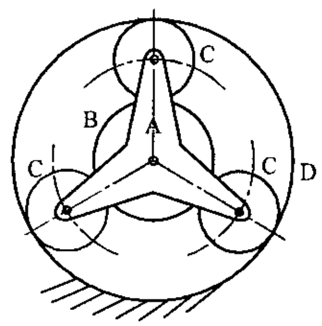

Fig. 7.22

Data :

$d = 228$ mm, $m = 4$ mm, $n_D = 0$, $n_A = 1$ rpm, $n_B = 5$ rpm, $M_{tB} = 30$ N m

Solution :

Number of teeth on D, $z_D = \dfrac{\text{Pitch circle diameter } d}{\text{Module } m} = \dfrac{228}{4} = 57$ teeth

Also $z_D = z_B + 2\,z_C$ or $z_C = \dfrac{z_D - z_B}{2}$ ......(1)

Condition	Arm - A	B - z_B	C - z_C	D - z_D
Fix the arm A. Give one clockwise revolution to B	0	+1	$-\frac{z_B}{z_C}\times 1$	$-\frac{z_C}{z_D}\times\frac{z_B}{z_C}$
Multiply by x	0	x	$-\frac{z_B}{z_C}x$	$-\frac{z_B}{z_D}x$
Add y	y	$y + x$	$y-\frac{z_B}{z_C}x$	$y-\frac{z_B}{z_D}x$

Speed of arm A = 1 revolution. From the third row of table, $y = 1$ ..... (2)
Speed of wheel B = 5 revolutions, $\therefore\ y + x = 5$ ..... (3)
Solving (2) and (3) we get, $x = 4$

The gear D is stationary, $\therefore\ y - \dfrac{z_B}{z_D}\, x = 0$

$$1 - 4\,\frac{z_B}{z_D} = 0$$

or $z_B = \dfrac{z_D}{4} = \dfrac{57}{4} = 14.25$ teeth

Since we cannot have quarter tooth, we have two possible values for z_B, either 15 or 14 teeth.

Let us try $z_B = 15$ teeth, $\therefore\ z_D = 4 \times 15 = 60$ teeth

from equation (1), $z_C = \dfrac{z_D - z_B}{2} = \dfrac{60-15}{2} = 22.5$ teeth

hence z_B = 15 teeth is inadmissible.

Try z_B = 14 teeth

Therefore, $z_D = 4 \times 14 = 56$ teeth

and $z_C = \dfrac{56-14}{2} = 21$ teeth

Hence number of teeth required are,

$$z_B = 14, z_C = 21 \text{ and } z_D = 56 \text{ teeth}$$

The correct pitch circle diameter of the ring,

$$D = z_D m = 56 \times 4 = 224 \text{ mm}$$

The energy equation is, $M_{tA} n_A + M_{tB} n_B + M_{tD} n_D = 0$

Output torque on gear B, $M_{tB} = -\dfrac{M_{tA} n_A}{n_B}$ $\quad (\because n_D = 0)$

$$= -\frac{30 \times 5}{1} = -150 \text{ N m}$$

The torque equation is, $M_{tA} + M_{tB} + M_{tD} = 0$

i.e., $30 - 150 + M_{tD} = 0$

Fixing torque on gear D, $M_{tD} = 120$ N m

Example 7.17

An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star shaped planet carrier C. The size of different tooth wheels are such that the planet carrier C rotates at 1/5th of the speed of the sun wheels . The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel is 100 N m. Determine;

(i) Number of teeth on different wheels of the train, and

(ii) Torque necessary to keep the internal gear stationary **(VTU - Jan 2006)**

Data :

$$n_E = 0, n_C = 1/5 \times n_S, z_S = 16 \text{ teeth}, M_{tS} = 100 \text{ N m}$$

Solution: *Refer fig. 7.23*

Number of teeth on E, $z_E = z_S + 2 z_P$

or $\quad z_P = \dfrac{z_E - z_S}{2}$ .... (1)

Condition	Carrier C (Arm)	Sun S - z_B	Planet P-z_p	Annular E-z_E
Fix the carrier C Give one clockwise revolution to B	0	1	$-\dfrac{z_S}{z_P} \times 1$	$-\dfrac{z_P}{z_E} \times \dfrac{z_S}{z_P}$
Multiply by x	0	x	$-\dfrac{z_S}{z_P}\, x$	$-\dfrac{z_S}{z_E}\, x$
Add y	y	$y + x$	$y - \dfrac{z_S}{z_P}\, x$	$y - \dfrac{z_S}{z_E}\, x$

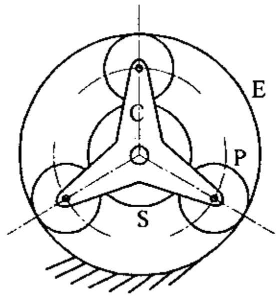

Fig. 7.23

Speed of the carrier C, $n_C = \frac{1}{5} \times n_S$

i.e., n_C = 1 revolution and n_S = 5 revolutions

Speed of C = 1 revolution $\therefore y = 1$ ..... (2)

Speed of S = 5 revolutions $\therefore y + x = 5$ ..... (3)

Solving the equations (2) and (3) we get, $x = 4$

The gear E is stationary, $\therefore n_E = y - \frac{z_S}{z_E} x = 0$

i.e., $1 - \frac{z_S}{z_E} \times 4 = 0$

$\therefore$ Number of teeth on E, $z_E = 4 \ z_S = 4 \times 16 = 64$

Number of teeth on P, $z_P = \frac{z_E - z_S}{2} = \frac{64 - 16}{2} = 24$

The energy equation is, $M_{tC} n_C + M_{tS} n_S + M_{tE} n_E = 0$

$\therefore$ Output torque on the carrier $M_{tC} = -\frac{M_{tS} n_S}{n_C}$ $(\because n_C = 0)$

$= -\frac{100 \times 5}{1} = -500$ N m

The torque equation is, $M_{tC} + M_{tS} + M_{tE} = 0$

i.e., $-500 + 100 + M_{tE} = 0$

$\therefore$ Fixing torque on gear E = 400 N m

Example 7.18

In an epicyclic gear train, the internal gears A, B and the compound gears C-D rotate independently about a common axis O. The gears E and F rotates on pins fixed to the arm G which turns independently about the axis O. E gears with A and C, F gears with B and D. All gears have the same module. The number of teeth on gears C, D, E and F are 28, 26, 18 and 18 respectively.

i) Sketch the arrangement.

ii) If the arm G makes 100 rpm clockwise and gear A is fixed, find the speed of gear B.

iii) If the arm G makes 100 rpm clockwise and gear A makes 10 rpm counter clockwise, find the speed of gear B. **(VTU - Jan 2005)**

Data: $z_C = 28$ teeth, $z_D = 26$ teeth, $z_E = 18$ teeth, $z_F = 18$ teeth

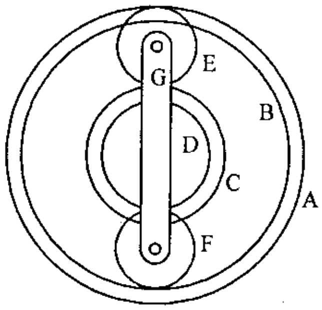

Fig. 7.24

Solution: *Refer fig. 7.24*

Since the module is the same for all gears, the number of teeth is proportional to the pitch circle radius.

∴ Number of teeth on A, $z_A = z_C + 2\,z_E = 28 + 2 \times 18 = 64$

Number of teeth on B, $z_B = z_D + 2\,z_F = 26 + 2 \times 18 = 62$

Condition	Arm-G	Compound Gear C-28	Compound Gear D-26	E-18	A-64	F-18	B-62
Fix the arm *G* Give one clockwise rotation to compond gear *C-D*	0	+1	+1	$-\frac{28}{18} \times 1$	$-\frac{18}{64} \times \frac{28}{18}$	$-\frac{26}{18} \times 1$	$\frac{18}{62} \times \frac{26}{18}$
Multiply by x	0	x	x	$-\frac{28}{18}\,x$	$-\frac{28}{64}\,x$	$\frac{26}{18}\,x$	$-\frac{26}{62}x$
Add y	y	$y + x$	$y + x$	$y - \frac{28}{18}\,x$	$y - \frac{28}{64}\,x$	$y - \frac{26}{18}\,x$	$y - \frac{26}{62}x$

ii) Arm G makes 100 rpm clockwise and the gear A is fixed.
From the last row of the table,

$$y = 100 \qquad .....(1)$$

and $$y - \frac{28}{64}\,x = 0 \qquad .....(2)$$

i.e., $$100 - \frac{28}{64}\,x = 0 \qquad \therefore\ x = 228.5714$$

Speed of gear B, $n_B = y - \frac{26}{62}\,x = 100 - \frac{26}{62} \times 228.5714 = 4.147$ rpm (clockwise)

iii) Arm G makes 100 rpm clockwise and the gear A makes 10 rpm counter clockwise.

From the last row of the table,

$$y = 100 \quad \text{..... (3)}$$

and $\quad y - \frac{28}{64}x = -10$

i.e., $\quad 100 - \frac{28}{64}x = -10 \quad \therefore \; x = 251.4286$

Speed of gear B, $\quad n_B = y - \frac{26}{62}x = 100 - \frac{26}{62} \times 251.4286$

$= -5.4378$ rpm (counter clockwise)

Example 7.19

An internal wheel B with 80 teeth is keyed to shaft F. A fixed internal wheel C with 82 teeth is concentric with B. A compound wheel DE gears with two internal wheels; D has 28 teeth and gears with C, while E gears with B. The compound wheel revolves freely on a pin which projects from a disc keyed to a shaft A co-axial with F. If all the wheels have the same pitch and shaft A makes 600 rpm, what is the speed of F? Sketch the arrangement. **(VTU-Aug. 2000)**

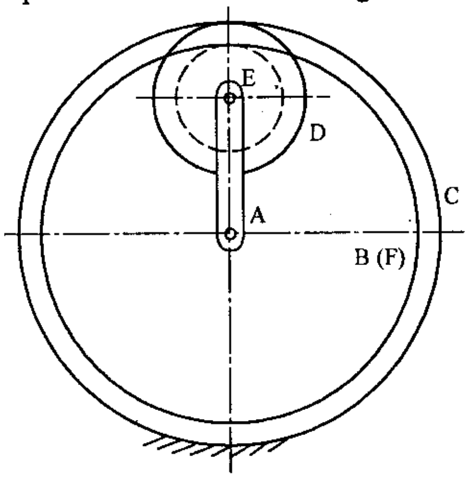

Fig. 7.25

Data :

$z_B = 80$ teeth, $z_c = 82$ teeth, $n_c = 0$ rpm, $z_D = 28$ teeth, $n_A = 600$ rpm

Solution : (Refer fig. 7.25)

Since the number of teeth is proportional to the pitch circle radius,

$$z_C - z_D = z_B - z_E$$

i.e, $\quad 82 - 28 = 80 - z_E$

$\therefore$ Number of teeth on gear E, $z_E = 26$ teeth

Condition	Arm - A	Compound gear		B - 80	C - 82
		D - 28	E - 26		
Fix the arm A Give one clockwise revolution to D	0	+1	+1	$\frac{26}{80}$	$\frac{28}{82}$
Multiply by x	0	x	x	$\frac{26}{80}x$	$\frac{28}{82}x$
Add y	y	$y+x$	$y+x$	$y+\frac{26}{80}x$	$y+\frac{28}{82}x$

Gear C is fixed. From the third row of the table, $y + \frac{28}{82}x = 0$ ..... (1)

Speed of arm A is 600 rpm, $\therefore$ y = 600 ..... (2)

Solving the equations (1) and (2) we get,

$$x = -1757.14 \quad \text{and} \quad y = 600$$

Speed of wheel B, $n_B = y + \frac{26}{80}x$

$$= 600 + \frac{26}{80} \times (-1757.14) = 28.93 \text{ rpm}$$

Example 7.20

An epicyclic gear train consists of a fixed annulus wheel A having 150 teeth. Meshing with A is a wheel B which drives wheel D through an idler wheel C. The wheel D being concentric with A. Wheels B and C are carried on an arm R which revolves clockwise at 100 rpm about the axis of A and D. If the wheels B and D have 25 and 40 teeth respectively, find the number of teeth on C and the speed and sense of rotation of C. **(VTU - Feb. 2002)**

Data :

$n_A = 0$, $z_A = 150$ teeth, $z_B = 25$ teeth, $z_D = 40$ teeth, $n_R = 100$ rpm

Solution : (Refer fig. 7.26)

The number of teeth is proportional to the pitch circle radius.

$\therefore$ $z_A = z_D + 2\,z_C + 2\,z_B$

i.e, $150 = 40 + 2\,z_C + 2 \times 25$

Number of teeth on C, $z_C = 30$ teeth

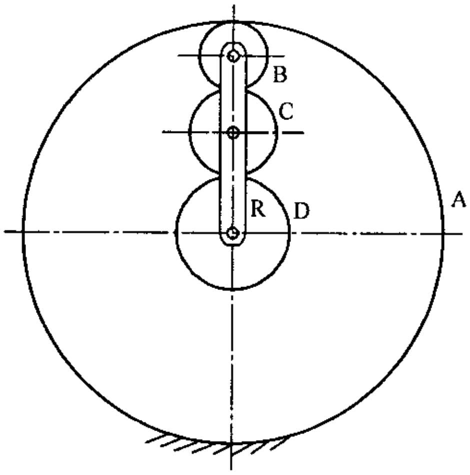

Fig. 7.26

Condition	Arm - R	D - 40	C - 30	B - 25	A - 150
Fix the arm R. Give one clockwise revolution to D	0	+1	$-\frac{40}{30} \times 1$	$\frac{30}{25} \times \frac{40}{30}$	$\frac{25}{150} \times \frac{30}{25} \times \frac{40}{30}$
Multiply by x	0	x	$-\frac{4}{3}x$	$\frac{8}{5}x$	$\frac{4}{15}x$
Add y	y	$y+x$	$y-\frac{4}{3}x$	$y+\frac{8}{5}x$	$y+\frac{4}{15}x$

Arm R rotates at 100 rpm. From the third row of the table, $y = 100$ ...... (1)

Gear A is fixed,

$$\therefore \quad y + \frac{4}{15}x = 0 \qquad \text{...... (2)}$$

Solving the equations (1) and (2) we get

$$x = -375 \quad \text{and} \quad y = 100$$

Speed of gear C, $n_c = y - \frac{4}{3}x$

$$= 100 - \frac{4}{3} \times (-375) = 600 \text{ rpm}$$

The direction of rotation of C is the same as that of the arm R (clockwise).

Example 7.21

A fixed annular wheel A and a smaller concentric rotating wheel B are connected by a compound wheel A_1B_1, the gear A_1 mesh with wheel A and B_1 with B. The compound wheel revolves on a pin on arm R which revolves about the axis of A and B. A has 130 teeth, B = 20, and B_1 = 80, the pitch of A and A_1 being twice the pitch of teeth B and B_1. How many revolutions will B make for one revolution of the arm R?

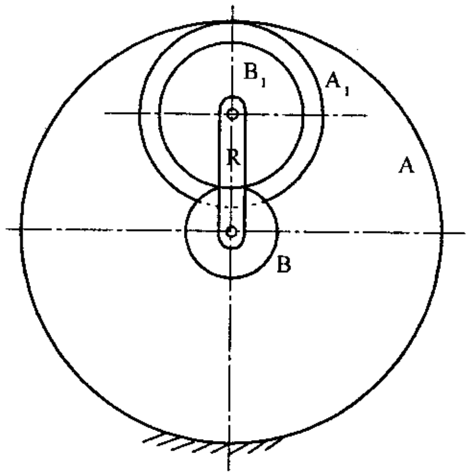

Fig. 7.27

Data :

$z_A = 130$ teeth, $z_B = 20$ teeth, $z_{B1} = 80$ teeth, $n_R = 1$ revolution

Solution : *(Refer fig. 7.27)*

As the number of teeth is proportional to pitch circle radius

$$\therefore \quad r_B + r_{B1} + r_{A1} = r_A$$

$$\text{or} \quad d_B + d_{B1} + d_{A1} = d_A$$

Also, the pitch circle diameter is proportional to the pitch module and the number of teeth. Let m be the module of gears B and B_1 and m_1 be the module of gears A and A_1.

$$\therefore z_B \times m + z_{B1} \times m + z_{A1} \times m_1 = z_A \times m_1$$

by data $p_1 = 2\,p$ or $m_1 = 2\,m$

Therefore the above equation becomes,

$$z_B \times m + z_{B1} \times m + z_{A1} \times 2\,m = z_A \times 2\,m$$

$$\text{or} \quad z_B + z_{B1} + 2\,z_{A1} = 2\,z_A$$

$$\text{i.e,} \quad 20 + 80 + 2\,z_{A1} = 2 \times 130$$

$\therefore$ The number of teeth on gear A_1, z_{A1} = 80 teeth

Condition	Arm R	B - 20	Compound gear		A - 130
			B_1 - 80	A_1 - 80	
Fix the arm R. Give one clockwise revolution to B	0	+1	$-\frac{20}{80}$	$-\frac{20}{80}$	$-\frac{80}{130}\times\frac{20}{80}$
Multiply by x	0	x	$-\frac{1}{4}x$	$-\frac{1}{4}x$	$-\frac{2}{13}x$
Add y	y	$y+x$	$y-\frac{1}{4}x$	$y-\frac{1}{4}x$	$y-\frac{2}{13}x$

Arm R makes one revolution. From the third row of the table, $y = 1$ ...... (1)

Gear A is fixed, i.e., $y - \frac{2}{13}x = 0$ ...... (2)

Substituting the value of y in equation (2) we get,

$$1 - \frac{2}{13}x = 0 \quad \text{or} \quad x = 6.5$$

Speed of gear B, $n_B = y + x = 1 + 6.5 = 7.5$ revolutions

The direction of rotation of gear B is the same as the arm R.

Example 7.22

In the epicyclic gear train shown in fig. 7.28, the driver wheel A has 14 teeth and the fixed annular wheel C has 100 teeth. The ratio of tooth numbers in wheels E and D is 98 : 41. If 2 kW at 1200 rpm is supplied to the wheel A, find the speed and direction of E, and the fixing torque required at C.

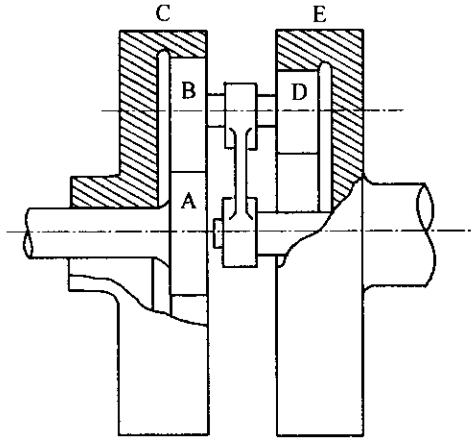

Fig. 7.28

Data :

$z_A = 14$ teeth, $z_C = 100$ teeth, $\frac{z_E}{z_D} = \frac{98}{41}$, $P = 2$ kW, $n_A = 1200$ rpm

Solution :

The number of teeth is proportional to the pitch circle radius

$\therefore \quad r_A + 2r_B = r_c$

or $\quad z_A + 2z_B = z_C$

i.e., $\quad 14 + 2z_B = 100$

Number of teeth on gear B, $z_B = 43$ teeth

Condition	Arm R	A-14	Compound gear		E-98	C-100
			B-43	D-41		
Fix the arm R. Give one clockwise revolution to A	0	+1	$-\frac{14}{43}\times 1$	$-\frac{14}{43}\times 1$	$-\frac{41}{98}\times\frac{14}{43}$	$-\frac{43}{100}\times\frac{14}{43}$
Multiply by x	0	x	$-\frac{14}{43}x$	$-\frac{14}{43}x$	$-\frac{41}{98}\times\frac{14}{43}x$	$-\frac{14}{100}x$
Add y	y	$y+x$	$y-\frac{14}{43}x$	$y-\frac{14}{43}x$	$y-\frac{41}{98}\times\frac{14}{43}x$	$y-\frac{14}{100}x$

Gear C is fixed, $\therefore\ y - \frac{14}{100}x = 0$ (1)

Gear A rotates at 1200 rpm,

$\therefore \quad y + x = 1200$ (2)

Solving the equations (1) and (2) we get,

$x = 1052.63$ and $y = 147.37$

Speed of gear E, $n_E = y - \frac{41}{98}\times\frac{14}{43}x = 147.37 - \frac{41}{98}\times\frac{14}{43}\times 1052.63 = 4$ rpm

The direction of rotation of gear E is the same as the gear A

Input torque on gear A, $M_{tA} = \frac{1000\,P}{\omega_A} = \frac{1000\,P\times 60}{2\pi\,n_A}$

$$= \frac{1000\times 2\times 60}{2\pi\times 1200} = 15.9 \text{ N m}$$

The energy equation is, $M_{tA}\, n_A + M_{tE}\, n_E + M_{tC}\, n_C = 0$

$\therefore$ Output torque on gear E, $M_{tE} = -\dfrac{M_{tA}\, n_A}{n_E}$ $\qquad (\because\ n_C = 0)$

$$= -\frac{15.9 \times 1200}{4} = -4770 \text{ N m}$$

The torque equation is, $M_{tA} + M_{tE} + M_{tC} = 0$

i.e., $15.9 - 4770 + M_{tC} = 0$

$\therefore$ Fixing torque on gear C, $M_{tC} = 4754.1$ N m

Example 7.23

An epicycle gear train is shown in fig. 7.29. The arm R rotates at 100 rpm and the gear A is fixed. Determine, (a) the speed of the shaft E, and (b) the torque on the fixed gear A, if the arm transmits 7.5 kW. The number of teeth on gears are: A = 25, B = 20, C = 30 and D = 75.

Data :

$n_R = 100$ rpm, $n_A = 0$, $P = 7.5$ kW, $z_A = 25$ teeth, $z_B = 20$ teeth, $z_C = 30$ teeth, $z_D = 75$ teeth

Solution :

Condition	**Arm R**	**A - 25**	**Compound gear**		**D(E) - 75**
			B - 20	**C - 30**	
Fix the arm R. Give one clockwise revolution to A	0	+1	$-\frac{25}{20} \times 1$	$-\frac{25}{20} \times 1$	$-\frac{30}{75} \times \frac{25}{20}$
Multiply by x	0	x	$-\frac{5}{4}x$	$-\frac{5}{4}x$	$-\frac{1}{2}x$
Add y	y	$y + x$	$y - \frac{5}{4}x$	$y - \frac{5}{4}x$	$y - \frac{1}{2}x$

Arm R rotates at 100 rpm. From the third row of the table, $y = 100$ ...... (1)

Gear A is fixed., $\therefore\ y + x = 0$ ...... (2)

Solving the equations (1) and (2) we get

$$x = -100 \quad \text{and} \quad y = 100$$

Speed of the shaft E, $n_E = y - \frac{1}{2}x = 100 - \frac{1}{2} \times (-100) = 150$ rpm

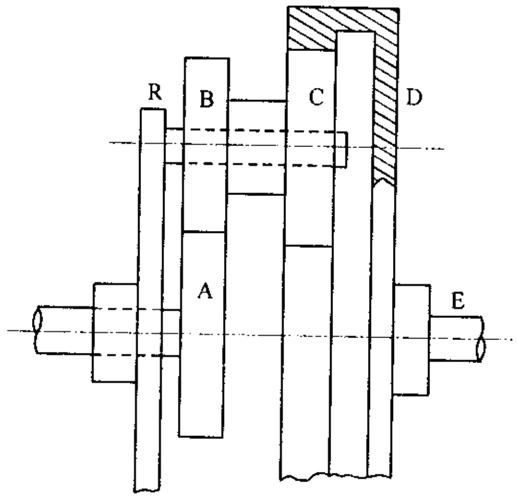

Fig. 7.29

The direction of rotation of E is the same as that of R

Input torque $M_{tR} = \dfrac{1000P}{\omega_R} = \dfrac{1000P \times 60}{2\,\pi\, n_R} = \dfrac{1000 \times 7.5 \times 60}{2\,\pi \times 100} = 716.19$ N m

The energy equation is, $M_{tR}\, n_R + M_{tE}\, n_E + M_{tA}\, n_A = 0$

∴ Output torque on gear E, $M_{tE} = -\dfrac{M_{tR}\, n_R}{n_E}$ $\quad (\because\ n_A = 0)$

$$= -\frac{716.19 \times 100}{150} = -477.46 \text{ N m}$$

The torque equation is, $M_{tR} + M_{tE} + M_{tA} = 0$

i.e., $716.19 - 477.46 + M_{tA} = 0$

∴ Fixing torque on gear A, $M_{tA} = -238.73$ N m

Example 7.24

In an epicyclic gear train shown in fig. 7.30, gear B is keyed to the shaft P and the gear E is keyed to the shaft Q. The gears C and D rotates together on a pin fixed to the arm A. The number of teeth on gears B, C and D are 20, 60 and 30 respectively. The shaft P rotates at 100 rpm and the shaft Q rotates at 240 rpm in the direction opposite to that of P. Assuming the module is same for all gears, find the speed and direction of the arm A.

Data:

$z_B = 20$ teeth, $z_C = 60$ teeth, $z_D = 30$ teeth, $n_P = n_B = 100$ rpm, $n_Q = n_E = -240$ rpm.

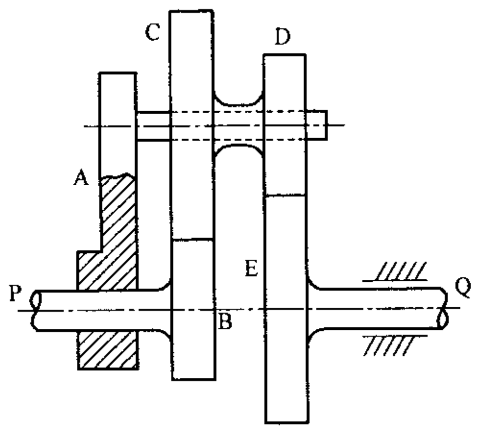

Fig. 7.30

Solution:

Since the module is same for all meshing gears, the number of teeth is proportional to the pitch circle radius. From the figure, the center distance of the two pairs of gears BC and ED must be same.

$$z_B + z_C = z_D + z_E$$

i.e., $20 + 60 = 30 + z_E$

$\therefore$ Number of teeth on gear E, $z_E = 50$

Condition	Arm - A	B - 20	Compound gear		E - 50
			C - 60	D - 30	
Fix the arm A Give one clockwise revolution to B	0	1	$-\frac{20}{60} \times 1$	$-\frac{20}{60} \times 1$	$\frac{30}{50} \times \frac{20}{60}$
Multiply by x	0	x	$-\frac{1}{3}x$	$-\frac{1}{3}x$	$\frac{1}{5}x$
Add y	y	$y + x$	$y - \frac{1}{3}x$	$y - \frac{1}{3}x$	$y + \frac{1}{5}x$

Gears B and E rotates 100 rpm and – 240 rpm respectively.

From the last row of the table,

$$y + x = 100 \quad \text{..... (1)}$$

and $$y + \frac{1}{5}x = -240 \quad \text{..... (2)}$$

(1) - (2) gives, $0.8\,x = 340$ $\therefore$ $x = 425$

Substitute the value of x in equation (1), we get

$$y + 425 = 100 \qquad \therefore\ y = -325$$

Speed of arm A, $n_A = y = -325$

The direction of arm A is same as the directio of shaft Q (gear E)

Example 7.25

An epicyclic gear train is shown in fig. 7.31. The annular gear A is held stationary and the compound gear BC is carried by an arm R. The number of teeth are: A = 72, B = 24, C = 12 and D = 18. If 5 kW power is delivered to D at 800 rpm with an efficiency of 94%, find the speed of the arm R. Also find the holding torque required on A. If the annulus A rotates at 100 rpm in the same direction as D, what will be the new speed of the arm R?

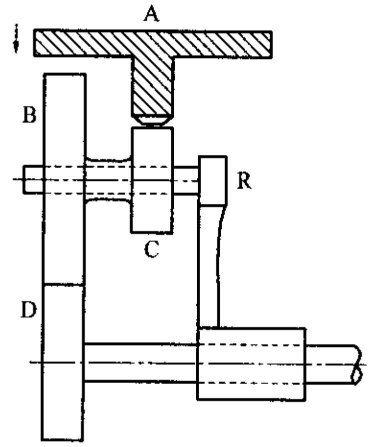

Fig. 7.31

Data:

$z_A = 72$ teeth, $z_B = 24$ teeth, $z_C = 12$ teeth, $z_D = 18$ teeth, $p = 5$ kW, $n_D = 800$ rpm, $\eta = 94\%$

Condition	Arm-R	B-24	Compound gear C-12	A-72	D-18
Fix the arm R Give one clockwise revolution to B - C	0	+1	+1	$\frac{12}{72} \times 1$	$-\frac{24}{18} \times \frac{20}{60}$
Multiply by x	0	x	x	$\frac{1}{6}x$	$-\frac{4}{3}x$
Add y	y	$y + x$	$y + x$	$y + \frac{1}{6}x$	$y - \frac{4}{3}x$

i) Gears A is fixed. From the third row of the table,

$$y + \frac{1}{6}x = 0 \qquad .....(1)$$

Speed of D is 800 rpm. i.e., $y - \frac{4}{3}x = 800$ .....(2)

Subtracting (2) from (1) gives, $1.5x = -800$

$$\therefore\ x = -533.33$$

From equation (1), $y - \frac{1}{6} \times 533.33 = 0 \qquad \therefore\ y = 88.888$

Speed of the arm R, $n_R = y = 88.888$ rpm (Direction is same as that of D)

Input torque to gear D, $M_{tD} = \frac{1000\,P}{\omega_D} = \frac{1000P \times 60}{2\pi n_D}$

$$= \frac{1000 \times 5 \times 60}{2\pi \times 800} = 59.683 \text{ N m}$$

The energy equation is

$$M_{tD}\,n_D + M_{tR}\,n_R + M_{tA}\,n_A = 0$$

$\therefore$ Output torque on the arm R, $M_{tR} = -\frac{M_{tD}n_D}{n_R} \times \eta$ $(\because\ n_A = 0)$

$$= -\frac{59.683 \times 800 \times 0.94}{88.888} = -504.923 \text{ N m}$$

The torque equation is, $M_{tD} + M_{tR} + M_{tA} = 0$

i.e., $59.683 - 504.923 + M_{tA} = 0$

$\therefore$ Holding torque $M_{tA} = 445.24$ N m

ii) Speed of A is 100 rpm. From the third row of the table,

$$y + \frac{1}{6}x = 100 \qquad ....(1)$$

Speed of D is 800 rpm. i.e., $y - \frac{4}{3}x = 800$ ....(2)

Solving the equations (1) and (2), we get

$$x = -466.67 \text{ and } y = 177.78$$

Speed of the arm R, $n_R = y = 177.78$ rpm

Example 7.26

An-epicyclic gear train is shown in fig. 7.32. The number of teeth on wheels D, E, F and G are 50, 20, 40 and 30 respectively. Wheel G is rigidly attached to shaft C and D to shaft A. Wheel D is kept stationary and the arm B is rotated at 300 rpm clockwise looking from A to C.

(a) Find the speed and direction of shaft C.

(b) If the shaft C develops 5 kW, what is the torque required to hold shaft A stationary? Neglect frictional losses. **(VTU - Jan 2003)**

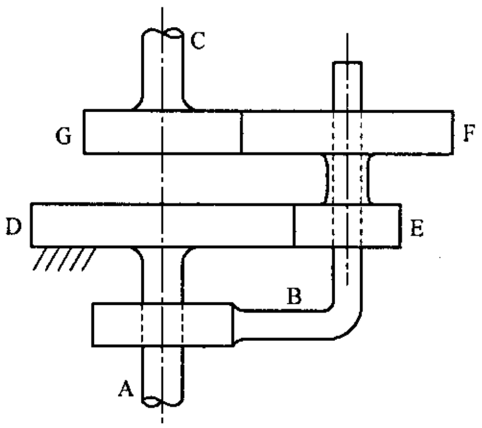

Fig. 7.32

Data: $Z_D = 50$ teeth, $Z_E = 20$ teeth, $Z_F = 40$ teeth, $Z_G = 30$ teeth, $P_C = 5$ kW

Solution:

Condition	Arm - B	G - 30	Compound gear F - 40	Compound gear E - 20	D - 50
Fix the arm B Give one clockwise revolution to G	0	+1	$-\frac{30}{40} \times 1$	$-\frac{30}{40} \times 1$	$\frac{20}{50} \times \frac{30}{40}$
Multiply by x	0	x	$-0.75\,x$	$-0.75\,x$	$0.3\,x$
Add y	y	$y+x$	$y-0.75\,x$	$y-0.75\,x$	$y+0.3\,x$

(a) Gear D is fixed and the arm B makes 300 rpm clockwise. From the last row of the table, we get

$$y = 300 \quad \text{..... (1)}$$

and $$y + 0.3\,x = 0 \quad \text{..... (2)}$$

i.e., $$300 + 0.3\,x = 0 \qquad \therefore \; x = -1000$$

Speed of gear G, $n_G = y + x = 300 - 1000 = -700$ rpm (counter clockwise).

(b) Input torque on gear G, $M_{tG} = \dfrac{1000\ P_G \times 60}{2\pi n_G} = \dfrac{1000 \times 5 \times 60}{2\pi \times 700} = 68.209$ N m

The energy equation is, $M_{tG} n_G + M_{tB} n_B + M_{tD} n_D = 0$

$\therefore$ Output torque $\quad M_{tB} = -\dfrac{M_{tG}\, n_G}{n_B} \qquad (\because\ n_D = 0)$

$= -\dfrac{68.209 \times (-700)}{300} = 159.154$ N m

The torque equation is, $M_{tG} + M_{tB} + M_{tD} = 0$

i.e., $68.209 + 159.154 + M_{tD} = 0$

Torque required to hold the shaft A (gear D), $M_{tD} = -227.363$ N m

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Example 7.27

Fig. 7.33 shows a planetary gear train that has two inputs. Sun gear S rotates at 500 rpm and the arm R rotates at 750 rpm, both clockwise as viewed from left. Determine the speed and direction of rotation of gear C.

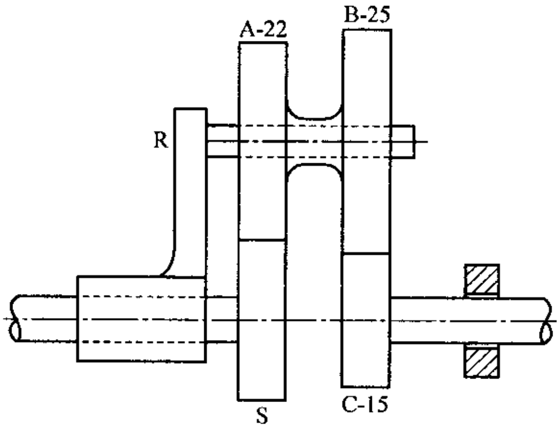


Fig. 7.33

Data:

$z_A = 22$ teeth, $z_B = 25$ teeth, $z_C = 15$ teeth, $n_S = 500$ rpm, $n_R = 750$ rpm

Solution:

From the fig. 7.33 the first and the last gears are on the same axis and the center distance between meshing gears axes are the same.

i.e., $\quad r_S + r_A = r_B + r_C$

Assuming the module is same for all gears, the number of teeth is proportional to pitch circle radius.

$z_S + z_A = z_B + z_C$

i.e., $\quad z_S + 22 = 25 + 15$

$\therefore$ Number of teeth on gear S, $z_S = 18$

Condition	Arm - R	S - 18	Compound gear A - 22	Compound gear B - 25	C - 15
Fix the arm *R* Give one clockwise revolution to gear S	0	+1	$-\frac{18}{22}\times 1$	$-\frac{18}{22}\times 1$	$-\frac{25}{15}\times\frac{18}{22}$
Multiply by *x*	0	x	$-\frac{9}{11}x$	$-\frac{9}{11}x$	$-\frac{15}{11}x$
Add *y*	y	$y+x$	$y-\frac{9}{11}x$	$y-\frac{9}{11}x$	$y+\frac{15}{11}x$

Arm rotates at 750 rpm. From the third row of the table,

$$y = 750 \qquad \text{..... (1)}$$

Gear S rotates at 500 rpm, $\therefore\ y + x = 500$ ..... (2)

From equations (1) and (2), $x = 500 - 750 = -250$

Speed of gear C, $n_C = y + \frac{15}{11}x = 750 + \frac{15}{11}\times(-250) = 409.09$ rpm

The direction of rotation is the same as that of S and R i.e., clockwise.

Example 7.28

An epicyclic speed reduction gear is shown in fig. 7.34. The driving shaft carries the arm A, a pin on which the compound wheel BC is free to revolve. Wheel C meshes with the fixed wheel E and wheel B meshes with wheel D which is keyed to the driven shaft. The number of teeth on the wheels are B = 40, C = 42, D = 45 and E = 43. Find the ratio of the speed of the driving shaft to the speed of the driven shaft. If the input torque to the driving shaft is 50 N m, calculate the output torque and the fixing torque on E. Also find the train value.

Data :

$z_B = 40$ teeth, $z_C = 42$ teeth, $z_D = 45$ teeth, $z_E = 43$ teeth, $M_{tA} = 50$ N m, $n_E = 0$

Condition	Arm - A	Compound gear B - 40	Compound gear C - 42	E - 43	D - 45
Fix the arm A. Give one clockwise revolution to B	0	+1	+1	$-\frac{42}{43}\times 1$	$-\frac{40}{45}\times 1$
Multiply by *x*	0	x	x	$-\frac{42}{43}x$	$-\frac{40}{45}x$
Add *y*	y	$y+x$	$y+x$	$y-\frac{42}{43}x$	$y-\frac{40}{45}x$

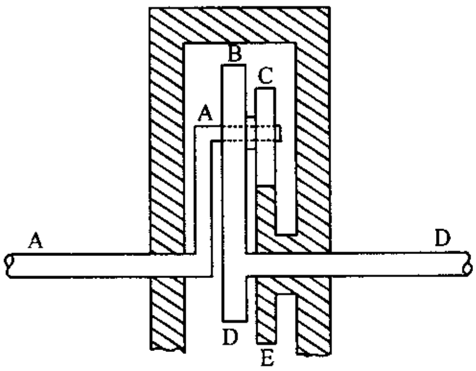


Fig. 7.34

The gear E is fixed, from the third row of table, $n_E = y - \frac{42}{43}x = 0$ ......(1)

Assume the input speed i.e., arm speed is 1 rpm

$$\therefore \; y = 1 \qquad ......(2)$$

$\therefore$ Substituting the value of $y = 1$ in equation (1) we get,

$$1 - \frac{42}{43}x = 0 \text{ or } x = \frac{43}{42}$$

The output shaft speed $n_D = y - \frac{40}{45}x$

$$= 1 - \frac{40}{45} \times \frac{43}{42} = 0.08995 \text{ rpm}$$

The energy equation is, $M_{tA}\, n_A + M_{tD}\, n_D + M_{tE}\, n_E = 0$

$\therefore$ Output torque on gear D, $M_{tD} = -\frac{M_{tA}\, n_A}{n_D}$ $(\because \; n_E = 0)$

$$= -\frac{50 \times 1}{0.08995} = -555.864 \text{ N m}$$

The torque equation is, $M_{tA} + M_{tD} + M_{tE} = 0$

i.e., $50 - 555.864 + M_{tE} = 0$

Fixing torque on gear E, $M_{tE} = 505.864$ N m

Train value $TV = \frac{n_D}{n_A} = \frac{0.08995}{1} = 0.08995$

Example 7.29

Fig.7.35 shows an epicyclic gear train. If shaft B makes 20 rpm in clockwise direction, find the speed of shaft C and its direction.

Data :

$n_B = 20$ rpm (clockwise), $z_2 = 50$, $z_3 = 120$, $z_4 = 32$, $z_5 = 29$, $z_6 = 80$, $z_7 = 40$, $z_8 = 55$, $z_9 = 30$, $z_{10} = 20$

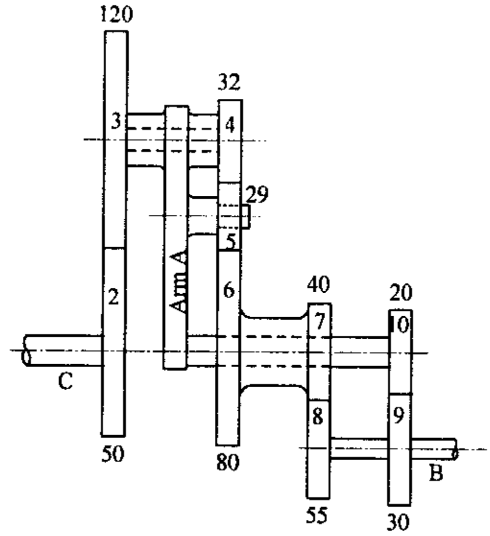


Fig. 7.35

Solution :

Speed of shaft B $= n_9 = n_8 = 20$ rpm

Consider gears 9 and 10,

$$n_{10} = \frac{z_9 n_9}{z_{10}} = \frac{30 \times 20}{20} = 30 \text{ rpm}$$

Since the gear 10 is mounted on the arm A, therefore the speed of the arm $n_A = -30$ rpm in anti-clockwise direction.

Similarly by considering gears 8 and 7,

$$n_7 = \frac{z_8 n_8}{z_7} = \frac{55 \times 20}{40} = 27.5 \text{ rpm, anti-clockwise direction}$$

since 6 - 7 is a compound gear,

$$n_6 = n_7 = -27.5 \text{ rpm}$$

Condition	Arm A	6 - 80	5 - 29	Compound gear		2 - 50
				4 - 32	3 - 120	
Fix the arm A. Give one clockwise revolution to 6	0	+1	$-\frac{80}{29}\times 1$	$\frac{29}{32}\times\frac{80}{29}$	$\frac{29}{32}\times\frac{80}{29}$	$-\frac{120}{50}\times\frac{29}{32}\times\frac{80}{29}$
Multiply by x	0	x	$-\frac{80}{29}x$	$\frac{5}{2}x$	$\frac{5}{2}x$	$-6x$
Add y	y	$y+x$	$y-\frac{80}{29}x$	$y+\frac{5}{2}x$	$y+\frac{5}{2}x$	$y-6x$

The arm A makes 30 rpm anti-clockwise direction.

$$\therefore y = -30 \quad \text{......(1)}$$

Speed of 6 is 27.5 rpm anti-clockwise,

$$\therefore y + x = -27.5 \quad \text{......(2)}$$

Solving the equations (1) and (2) we get, $x = 2.5$

$$\text{Speed of C} = n_2 = y - 6x$$

$$= -30 - 6 \times 2.5 = -45 \text{ rpm (anti-clockwise)}$$

Example 7.30

The shaft A shown in fig 7.36 rotates at 300 rpm and the shaft B rotates at 600 rpm, in the direction opposite to that of the shaft A. Determine the speed and the direction of rotation of shaft C.

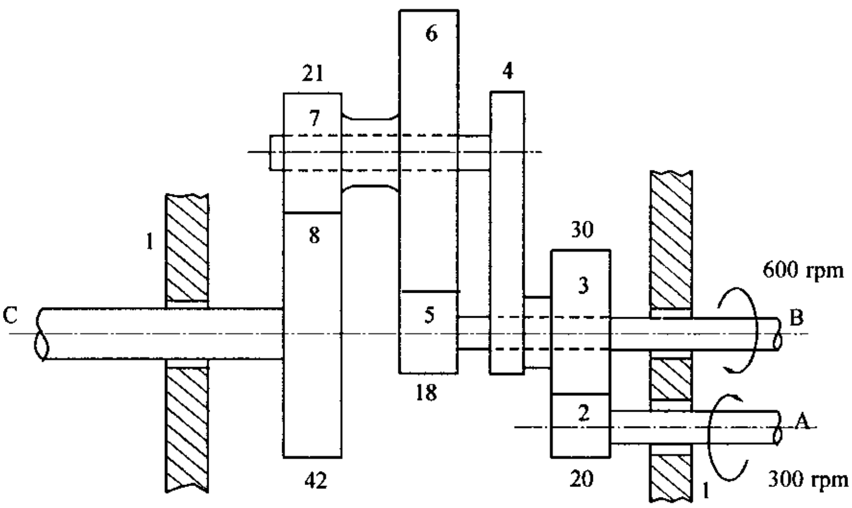


Fig. 7.36

Data :

$n_A = n_2 = 300$ rpm, $n_B = n_5 = -600$ rpm, $z_2 = 20$ tech,
$z_3 = 30$ teeth, $z_5 = 18$ teeth, $z_6 = 45$ teeth, $z_7 = 21$ teeth, $z_8 = 42$ teeth
Considering the gears 2 and 3 (arm 4)

Speed of the gear 3 (arm 4) $n_3 = \frac{z_2}{z_3} \times n_2 = \frac{20}{30} \times 300 = 200$ rpm

The direction of rotation of gear 3 is opposite to that of gear 2.

$\therefore \quad n_3 = -200$ rpm (anti-clockwise)

Condition	Arm - 4	5 - 18	Compound gear		8 - 42
			6 - 45	7 - 21	
Fix the arm 4. Give one clockwise revolution to 5	0	1	$-\frac{18}{45}$	$-\frac{18}{45}$	$\frac{21}{42} \times \frac{18}{45}$
Multiply by x	0	x	$-\frac{18}{45}x$	$-\frac{18}{45}x$	$\frac{21}{42} \times \frac{18}{45}x$
Add y	y	$y+x$	$y-\frac{18}{45}x$	$y-\frac{18}{45}x$	$y+\frac{21}{42} \times \frac{18}{45}x$

Arm 4 rotates at 200 rpm counter clockwise direction

$\therefore \quad y = -200 \quad$ ..... (1)

Gear 5 rotates 600 rpm counter clockwise

$\therefore \quad y + x = -600 \quad$ ..... (2)

Solving the equations (1) and (2) we get,

$x = -400$ and $y = -200$

Speed of shaft C, $n_8 = y + \frac{21}{42} \times \frac{18}{45} x$

$= -200 + \frac{21}{42} \times \frac{18}{45} \times (-400) = -280$ rpm

The direction of rotation is opposite to that of shaft A.

Example 7.31

In an epicyclic gear train shown in fig. 7.37 the gear C has teeth cut both internally and externally and is free to rotate on an arm driven by the shaft S_1. The gear C meshes externally with the casing D and internally with the pinion B. The gears have the following number of teeth, $z_B = 24$, $z_C = 32$ and 40, $z_D = 48$. Determine the velocity ratio between;

(i) S_1 and S_2 when the casing D is fixed

(ii) S_1 and D when the shaft S_2 is fixed.

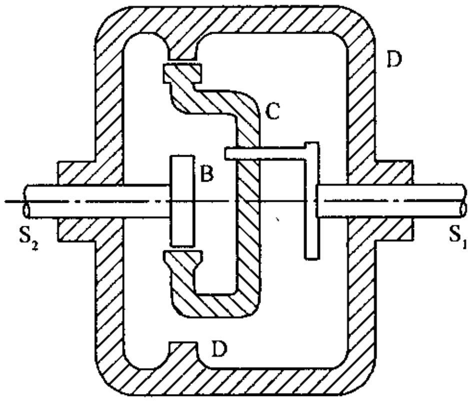


Fig. 7.37

Data : $z_B = 24,\ z_{CI} = 32,\ z_{CE} = 40,\ z_D = 48$

Solution :

Condition	Arm S_1	B - 24	Gear C		D - 48
			C_I - 32	C_E - 40	
Fix the arm S_1. Give one clockwise revolution to B	0	1	$\frac{24}{32} \times 1$	$\frac{24}{32} \times 1$	$\frac{40}{48} \times \frac{24}{32}$
Multiply by x	0	x	$0.75\,x$	$0.75\,x$	$0.625\,x$
Add y	y	$y + x$	$y + 0.75\,x$	$y + 0.75\,x$	$y + 0.625\,x$

(i) Casing D is fixed.

$\therefore \quad y + 0.625\,x = 0$

or $\quad x = -1.6\,y$

Velocity ratio of S_1 and S_2, $\dfrac{n_{S1}}{n_B} = \dfrac{y}{y+x} = \dfrac{y}{y-1.6y} = -1.667$

(ii) Shaft S_2 (gear B) is fixed

$\therefore \quad y + x = 0$

or $\quad x = -y$

Velocity ratio of S_1 and D, $\dfrac{n_{S1}}{n_D} = \dfrac{y}{y+0.625x} = \dfrac{y}{y-0.625y} = 2.667$

Example 7.32

In the hoisting mechanism shown in fig. 7.38, A is a fixed annular, having 100 teeth. The two pinions P are carried by the arm R of the epicyclic train, which also carries the drum B. Gear S, which is attached to the crank has 50 teeth. The diameter of the drum is 0.3 m, crank radius is 0.5 m, and the force applied at crank is 200 N. Find the number of teeth on the pinions P and weight lifted, neglecting the effect due to starting and friction.

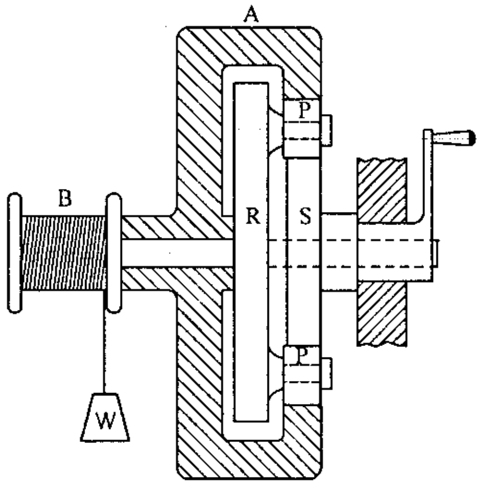


Fig. 7.38

Data :

$z_A = 100$ teeth, $z_S = 50$ teeth, $D = 0.3$ m, $R = 0.15$ m, $F = 200$ N, $r = 0.5$ m

Solution :

Since the diameteral pitch for meshing gears are same,

$$z_A = z_S + 2\,z_P$$

$$\therefore z_P = \frac{z_A - z_S}{2} = \frac{100-50}{2} = 25 \text{ teeth}$$

Condition	Arm - R	S - 50	P - 25	A - 100
Fix the arm R. Give one clockwise revolution to S.	0	+1	$-\frac{50}{25}\times 1$	$-\frac{25}{100}\times\frac{50}{25}$
Multiply by x	0	x	$-2\,x$	$-\frac{1}{2}x$
Add y	y	$y + x$	$y - 2\,x$	$y - \frac{1}{2}x$

Annular wheel A is fixed,

$$\therefore \quad y - \frac{1}{2}x = 0 \qquad \text{......(1)}$$

Assume the speed of S (driver) = 1 revolution

$$\therefore y + x = 1 \qquad \text{......(2)}$$

Solving the above equations, we get

$$x = \frac{2}{3} \text{ and } y = \frac{1}{3}$$

$$\therefore \text{Speed of arm} = \text{speed of B} = y = \frac{1}{3}$$

$$\text{Torque on S, } M_{tS} = \text{Force applied} \times \text{crank radius} = F \times r$$

$$= 200 \times 0.5 = 100 \text{ N m}$$

by energy equation

$$M_{tS}\, n_S = M_{tR}\, n_R$$

or

$$M_{tR} = \frac{M_{tS}\, n_S}{n_R} = \frac{100 \times 1}{(1/3)} = 300 \text{ N m}$$

but torque on arm M_{tR} = Weight lifted × Radius of drum = WR

$$\text{i.e., } 300 = W \times 0.15$$

$$\therefore \text{Weight to be lifted } W = \frac{300}{0.15} = 2000 \text{ N}$$

Example 7.33

The Humpage's reduction gear is shown in fig. 7.39. The input is to gear A, and the output is gear D which is connected to the output shaft. The arm R turns freely on the output shaft and carries the compound gear BC. Gear E is fixed to the frame. The number of teeth on the wheel A, B, C, D and E are respectively 20, 56, 24, 35 and 76. If gear A receives 3.75 kW at 1000 rpm, determine the output torque and the train value.

Data :

$z_A = 20$ teeth $z_B = 56$ teeth, $z_C = 24$ teeth, $z_D = 35$ teeth, $z_E = 76$ teeth, $n_A = 1000$ rpm, $P = 3.75$ kW

Solution :

Wheel E is fixed, $\quad \therefore \quad n_E = y - \frac{5}{19}x = 0 \qquad$ ......(1)

Speed of A is 1000 rpm, $\therefore\ y + x = 1000 \qquad$ ......(2)

Substracting (1) from (2),

$$\left(1 + \frac{5}{19}\right)x = 1000$$

or

$$x = 791.67$$

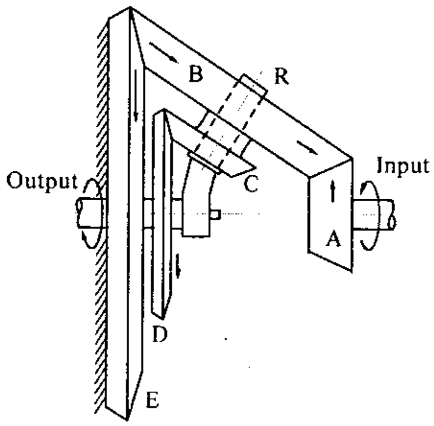


Fig. 7.39

Condition	Arm R	A -20	Compound gear		D - 35	E - 76
			B- 56	C- 24		
Fix the arm R. Give one clockwise revolution to A.	0	+1	$\frac{20}{56} \times 1$	$\frac{20}{56} \times 1$	$-\frac{24}{35} \times \frac{20}{56}$	$-\frac{56}{76} \times \frac{20}{56}$
Multiply by x	0	x	$\frac{5}{14}x$	$\frac{5}{14}x$	$-\frac{12}{49}x$	$-\frac{5}{19}x$
Add y	y	$y+x$	$y+\frac{5}{14}x$	$y+\frac{5}{14}x$	$y-\frac{12}{49}x$	$y-\frac{5}{19}x$

Substituting the value of x in equation (2) we get,

$$y + 791.67 = 1000$$

$$\therefore \quad y = 1000 - 791.67 = 208.33$$

Speed of D, $n_D = y - \frac{12}{49}x$

$$= 208.33 - \frac{12}{49} \times 791.67 = 14.45 \text{ rpm}$$

Input torque on the gear A, $M_{tA} = \frac{1000P}{\omega_A} = \frac{1000P \times 60}{2\pi n_A}$

$$= \frac{1000 \times 3.75 \times 60}{2\pi \times 1000} = 35.8 \text{ N m}$$

The energy equation is, $M_{tA}n_A + M_{tD}n_D + M_{tE}n_E = 0$

$\therefore$ Output torque on gear D, $M_{tD} = -\dfrac{M_{tA}n_A}{n_D}$ $\qquad (\because\ n_E = 0)$

$$= -\frac{35.8 \times 1000}{14.45} = -2477.5 \text{ N m}$$

The torque equation is, $M_{tA} + M_{tD} + M_{tE} = 0$

i.e., $35.8 - 2477.5 + M_{tE} = 0$

$\therefore$ Fixing torque on gear E, $M_{tE} = 2441.7$ N m

Bevel gear differential

Fig. 7.40 shows a bevel gear differential as used in automobiles. Gear A is driven by the engine and drives the hypoid ring gear B which turns loosely on the hub of gear D, which is keyed to the axle of the left wheel. Gear E is keyed to the axle of the right wheel. The gear B is fastened to the carrier R in which the planet wheels C are freely mounted. Gears C meshes with gear D and gear E. When the car is travelling in a straight line, gears C, D and E turn as a unit with the carrier and there is no relative motion between them. When the car makes a turn, the outside rear wheel must increase in speed and the inner wheel must decrease in speed by the same amount in order that the tyres will not slip. During a turn, the planet gears C rotate about their own axes, thus permitting gears D and E to revolve at different velocities. It is interesting to note that the sum of the axle speeds is always twice the ring gear speed.

Thus, $\qquad \omega_D + \omega_E = 2\ \omega_B$

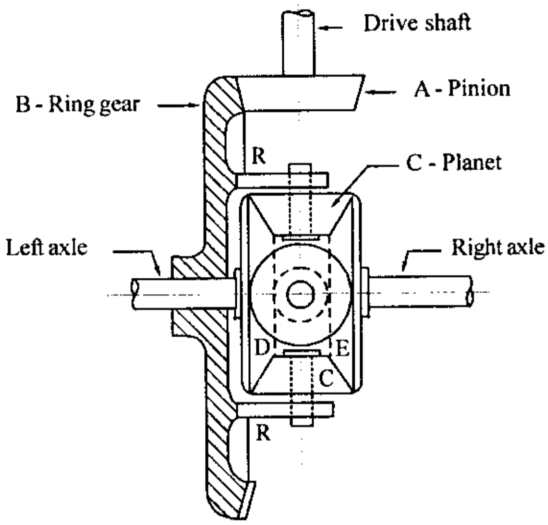


Fig. 7.40

The total torque on the carrier R is twice the torque on either axle. When one wheel is on ice or on slippery surface, the friction force between the tire and the slippery surface is very small and, therefore the torque on that axle is very small. Since the axle torques must be equal, the force available at the other wheel must be equal to that for the wheel on slippery surface. Therefore the automobile may not be able to move under its own power when one of the driving wheel is on slippery surface.

Example 7.34

Fig. 7.40 shows a differential gear used for gear axle in an automobile. While negotiating a curve, the road wheel which is driven by E has a speed of 220 rpm. Find the speed of the road wheel D, if the driving shaft pinion A rotates at 1000 rpm. The number of teeth on gears A and B are 12 and 60 respectively.

Data :

$n_E = 220$ rpm, $n_A = 1000$ rpm, $z_A = 12$ teeth, $z_B = 60$ teeth

Solution :

Consider the gears A and B, $\frac{z_A}{z_B} = \frac{n_B}{n_A}$

$\therefore$ Speed of the gear B, $n_B = \frac{z_A n_A}{z_B} = \frac{12 \times 1000}{60} = 200$ rpm

We know that, $\omega_D + \omega_E = 2\,\omega_B$

or $n_D + n_E = 2\,n_B$

$\therefore$ Speed of road wheel D, $n_D = 2\,n_B - n_E$

$= 2 \times 200 - 220 = 180$ rpm

REVIEW QUESTIONS

1. What do you understand by gear train?
2. Explain the train value. How is it related to velocity ratio?
3. Name different types of gear trains and give examples where each of these types is used in practice.
4. Describe with neat sketches the following gear train :
   (i) Simple, (ii) Compound, (iii) Reverted, and (iv) Epicyclic gear train.
5. Describe the bevel gear differential with neat sketch.

EXERCISE - 7

1. In the epicyclic gear train shown in fig. 7.41, gear B and C are keyed to a shaft carried on a revolving arm R. Arm R turns about the axis of gears A and D. If all the gear teeth have the same pitch, find the number of teeth on D. If D rotates at 600 rpm clockwise, find the speed and sense of rotation of arm R. [**Ans.** 18 teeth, 36.3 rpm clockwise]

(VTU - July 2004)

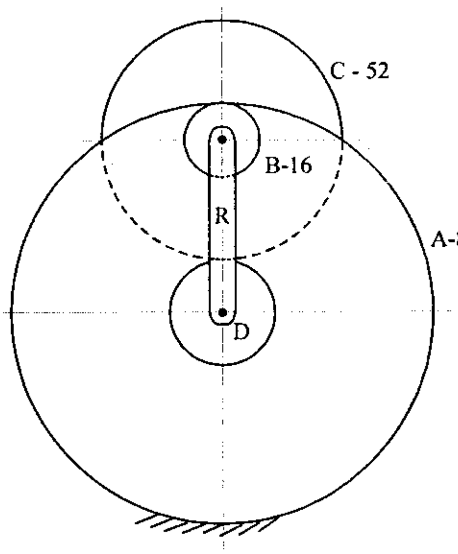


Fig. 7.41

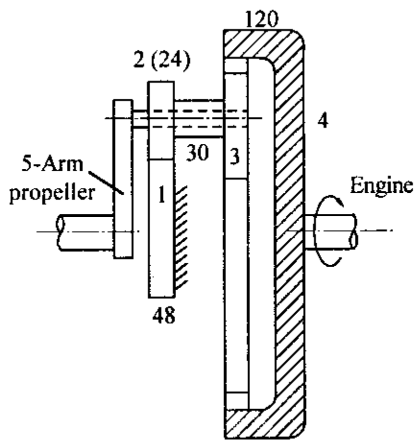


Fig. 7.42

2. An aircraft propeller reduction drive is shown in fig 7.42. Determine the propeller speed in magnitude and direction if the engine runs at 1500 rpm. The gears 1, 2, 3 and 4 have 48, 24, 30 and 120 teeth respectively. [**Ans.** 1000 rpm, direction is same as that of engine]

3. In an epicyclic gear train, wheel A is keyed on the driving shaft. Wheel B gears with A and also with a fixed annulus wheel C. Wheels B and D are fixed to the common spindle which is carried by an arm which can rotate about the axes of the wheel A and the wheel D gears with an annular wheel E which is keyed to the driven shaft. If A has 20, B –24 and D - 16 teeth and all the teeth have the same pitch, find the velocity ratio of the two shafts.

[**Ans.** 18]

4. For the gear train shown in fig. 7.43 shaft A rotates at 300 rpm and the shaft B at 600 rpm in the same sense of A. Determine the speed and the direction of rotation of shaft C.

[**Ans.** 1281.82 rpm]

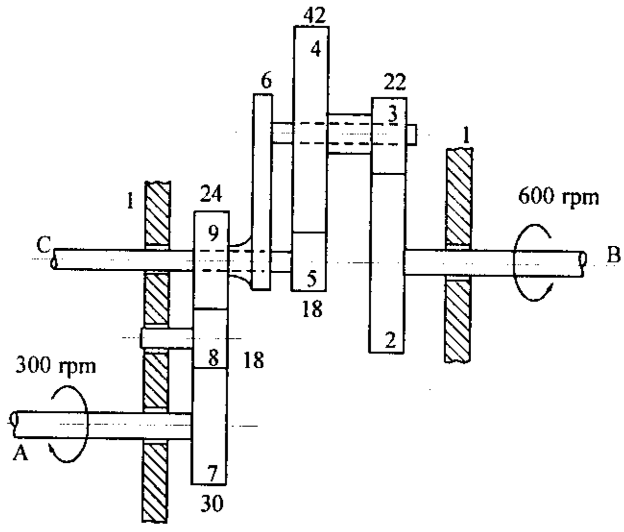


Fig. 7.43

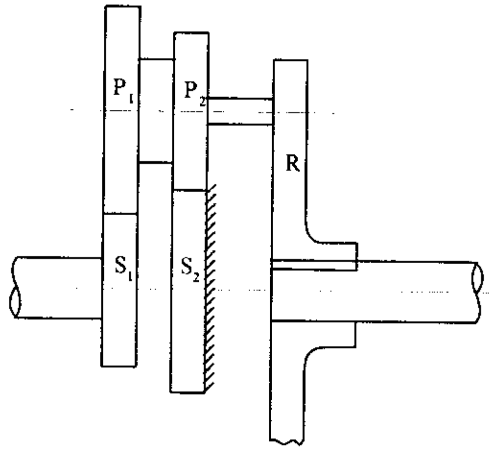


Fig. 7.44

5. An epicycle gear shown in fig. 7.44 consists of two sun wheels S_1 and S_2 with 24 and 36 teeth respectively, engaged with the compound planet wheels P_1, P_2 and P_1 has 40 teeth. S_1 is keyed to the driving shaft which is co-axial with the driven shaft. Find the velocity ratio of the train. If 7.5 kW is transmitted at 1000 rpm, what torque is required to hold S_2.
[**Ans.** 2, 71.62 N-m]

6. An epicyclic gear train is shown in fig. 7.45. The sun wheel S has 15 teeth and is fixed to the motor shaft rotating at 1450 rpm. The planet P has 45 teeth, gears with fixed annulus A and rotates on a spindle carried by an arm which is fixed to the output shaft. The planet P also gears with the sun wheel S. Find the speed of the output shaft. If the motor is transmitting 1.5 kW, find the torque required to fix the annulus. [**Ans.** 181.25 rpm, 70 N-m]

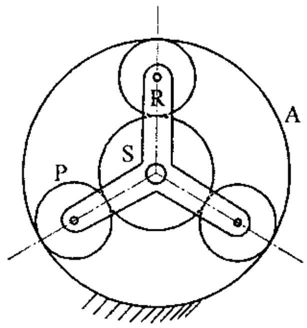


Fig. 7.45

7. The fig. 7.46 shows an epicyclic gear train. Pinion A has 15 teeth and is rigidly fixed in the motor shaft. The wheel B has 20 teeth and gears with A and also with annular fixed wheel D. Pinion C has 15 teeth and is integral with B (C and B being a compound wheel). Gear C meshes with annular wheel E, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B-C. If the motor runs at 100 rpm, find the speed of the machine shaft. Find also the torque exerted on the machine shaft if the motor develops a torque of 100 N-m.

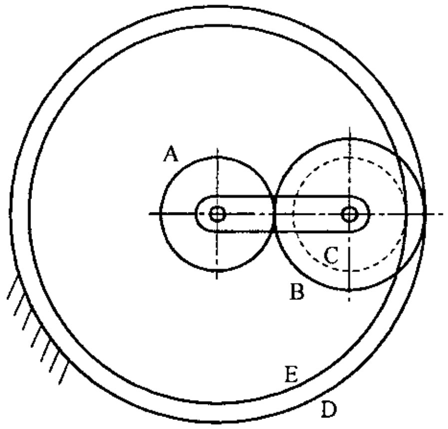


Fig. 7.46

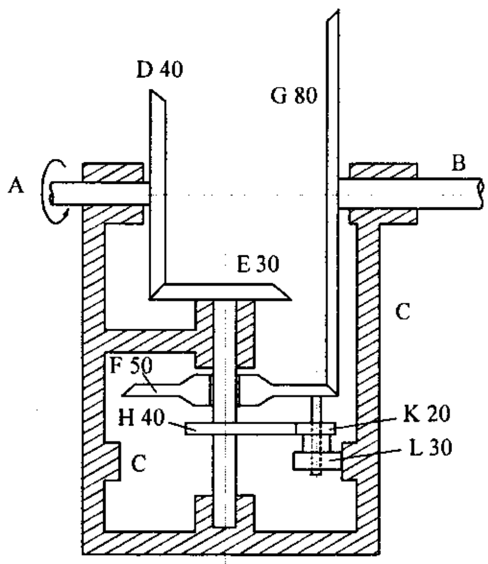


Fig. 7.47

7. In the gear train shown in fig. 7.47, A is the driving shaft which rotates at 270 rpm in the anti clockwise direction (viewed from left). The casing (which is an annular wheel C) is fixed. Gear E and H are keyed to the vertical shaft on which F is free to rotate. Compound gears K and L are rigidly connected to the pin carried by F. The number of teeth on each gear is indicated in the figure. Determine the speed at the output shaft B. [**Ans.** 90 rpm]

8. In the epicyclic gear train shown in fig. 7.48, gear A rotates at 1000 rpm clockwise, while E rotates at 500 rpm anti-clockwise direction. Determine the speed and direction of the annulus D, and the shaft F. All gears are of the same pitch, and the number of teeth in A is 30, B is 20 and E is 80. [**Ans.** – 372 rpm, 40 rpm]

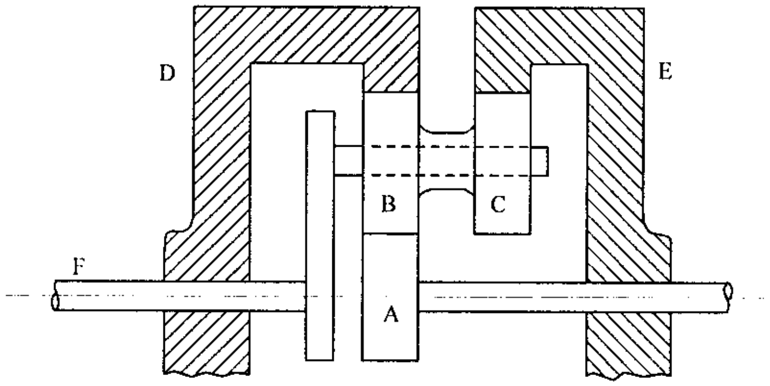


Fig. 7.48

9. (a) An epicyclic gear train is shown in fig. 7.10. The number of teeth on wheels 2,3,4 and 5 are 50, 20, 40 and 30 respectively. Gear 2 is kept stationary and arm A is rotated at 300 rpm clockwise. Find the speed and direction of gear 5.

   (b) If the gear 5 develops 5 kW, what is the torque required to hold arm stationary? Neglect frictional losses. **[Ans.** – 700 rpm, –90.45 N-m]

10. In an epicyclic gear train of sun and planet type, the pitch circle diameter of the annular wheel A is to be nearly equal to 220 mm and the module is 4 mm. When the annular wheel is stationary, the spider which carries three planet gears P and equal size has to make one revolution for every five revolution of the driving spindle carrying sun wheel S. Determine the number of teeth on all the wheels and also the exact pitch diameter of the wheel A.

    (VTU - July 2006) **[Ans.** $z_C = 56,\ z_P = 21, z_S = 21, z_S = 14, d_A = 224$ mm)

11. In the epicyclic gear train shown in fig. 7.19, wheels A, D and E are free to rotate on the axis P. The compound wheel BC rotate on the axis Q at the end of the arm R. All the gears have equal pitch. The number of external teeth on gears A, B and C are 12, 30 and 14 respectively. The gears D and E are annular gears. The gear A rotates at 60 rpm in the clockwise direction and the gear D rotates at 300 rpm counter clockwise. Determine the speed and direction of the arm R and the gear E. **(VTU - July 2003)**

    [Ans. –267.27 rpm, – 321.815 rpm]

12. A fixed annular gear A and a smaller concentric rotating gear B are connected by a compound wheel C-D. The gear C mesh with gear A and D with B. The compound gear revolves on a pin on the arm R which revolves about the axis of A and B. The number of teeth on gears A, B and D are 150, 40 and 100 respectively. Determine the number of teeth on gear C if the gears A and C being twice the module of gears B and D. How many revolutions will B make for one complete revolution of arm R ? **(VTU - July 2007)**

    [Ans. 80, 5.6875 revolutions]

8

CAMS

Introduction

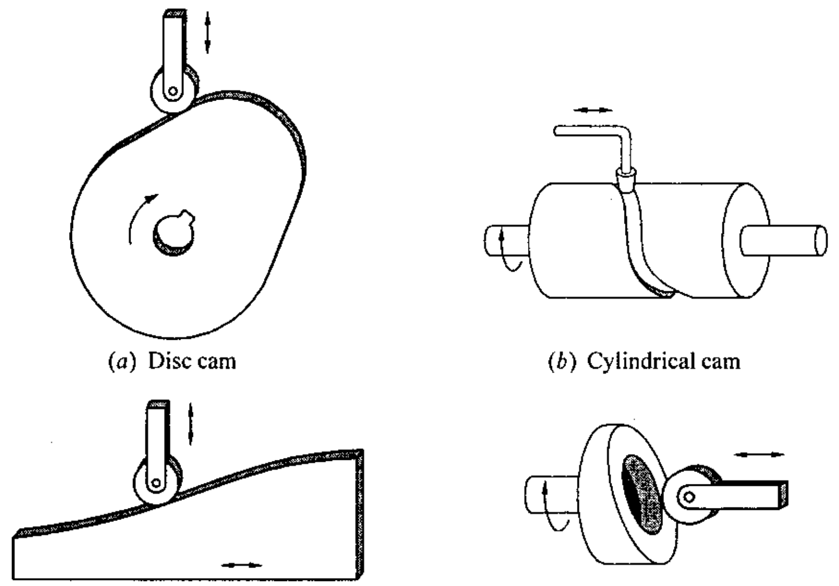
A *cam* is a mechanical member for transmitting a desired motion to a follower by direct contact. The simplest cam mechanism consist of a cam, a follower and a frame. The cam accepts an input motion similar to a crank and imparts a resultant motion to a follower.

The unique feature of a cam is that it can impart a very distinct motion to its follower. Cams can be used to obtain irregular motion that would be difficult to obtain from other linkages. Cams are used in automotive engines and automation equipments.

The disadvantages of cams are manufacturing cost, poor wear resistance and relatively poor high speed capability.

Types of cams

Cams are classified according to their basic shapes. The different types of cams are: disc cam, cylindrical cam, wedge cam, and face cam.



(*a*) Disc cam (*b*) Cylindrical cam

(*c*) Wedge cam (*d*) Face cam

Fig. 8.1